

Chapter
10

Quadratic and Exponential Functions

What You'll Learn

- **Lesson 10-1** Graph quadratic functions.
- **Lessons 10-2 through 10-4** Solve quadratic equations.
- **Lesson 10-5** Graph exponential functions.
- **Lesson 10-6** Solve problems involving exponential growth and exponential decay.
- **Lesson 10-7** Recognize and extend geometric sequences.

Why It's Important

Quadratic functions and equations are used to solve problems about fireworks, to simulate the flight of golf balls in computer games, to describe arches, to determine hang time in football, and to help with water management. Exponential functions are used to describe changes in population, to solve compound interest problems, and to determine concentration of chemicals in a body of water after a spill. Exponential decay is one type of exponential function. Carbon dating uses exponential decay to determine the age of fossils and dinosaurs. *You will learn about carbon dating in Lesson 10-6.*

Key Vocabulary

- parabola (p. 524)
- completing the square (p. 539)
- Quadratic Formula (p. 546)
- exponential function (p. 554)
- geometric sequence (p. 567)



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 10.

For Lesson 10-1

Graph Functions

Use a table of values to graph each equation. (For review, see Lesson 5-3.)

- $y = x + 5$
- $y = 2x - 3$
- $y = 0.5x + 1$
- $y = -3x - 2$
- $2x - 3y = 12$
- $5y = 10 + 2x$
- $x + 2y = -6$
- $3x = -2y + 9$

For Lesson 10-3

Perfect Square Trinomials

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

(For review, see Lesson 9-6.)

- $t^2 + 12t + 36$
- $a^2 - 14a + 49$
- $m^2 + 18m - 81$
- $y^2 + 8y + 12$
- $9b^2 - 6b + 1$
- $6x^2 + 4x + 1$
- $4p^2 + 12p + 9$
- $16s^2 - 24s + 9$

For Lesson 10-7

Arithmetic Sequences

Find the next three terms of each arithmetic sequence. (For review, see Lesson 4-7.)

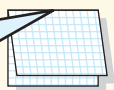
- 5, 9, 13, 17, ...
- 12, 5, -2, -9, ...
- 4, -1, 2, 5, ...
- 24, 32, 40, 48, ...
- 1, -6, -11, -16, ...
- 27, -20, -13, -6, ...
- 5.3, 6.0, 6.7, 7.4, ...
- 9.1, 8.8, 8.5, 8.2, ...

FOLDABLES™ Study Organizer

Make this Foldable to help you organize information on quadratic and exponential functions. Begin with four sheets of grid paper.

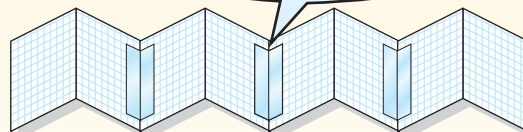
Step 1 Fold in Half

Fold each sheet in half along the width.



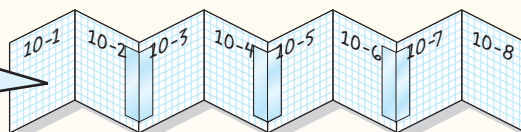
Step 2 Tape

Unfold each sheet and tape to form one long piece.



Step 3 Label

Label each page with the lesson number as shown. Refold to form a booklet.



Reading and Writing As you read and study the chapter, write notes and examples for each lesson on each page of the journal.

What You'll Learn

- Graph quadratic functions.
- Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola.

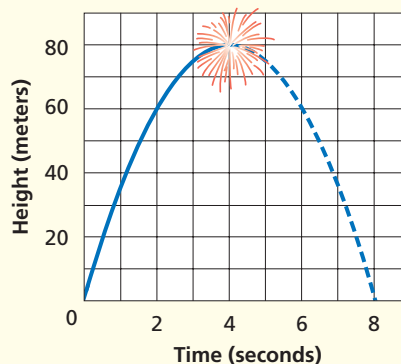
Vocabulary

- quadratic function
- parabola
- minimum
- maximum
- vertex
- symmetry
- axis of symmetry

How can you coordinate a fireworks display with recorded music?

The Sky Concert in Peoria, Illinois, is a 4th of July fireworks display set to music. If a rocket (firework) is launched with an initial velocity of 39.2 meters per second at a height of 1.6 meters above the ground, the equation $h = -4.9t^2 + 39.2t + 1.6$ represents the rocket's height h in meters after t seconds. The rocket will explode at approximately the highest point.

Height of Rocket

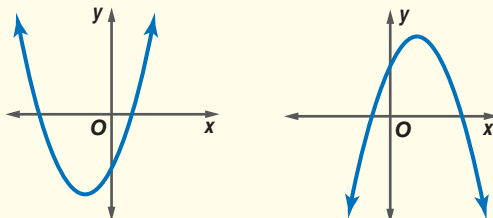


GRAPH QUADRATIC FUNCTIONS The function describing the height of the rocket is an example of a quadratic function. A **quadratic function** can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form of the quadratic function is called the *standard form*. Notice that this polynomial has degree 2 and the exponents are positive. The graph of a quadratic function is called a **parabola**.

Key Concept

Quadratic Function

- **Words** A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.
- **Models**

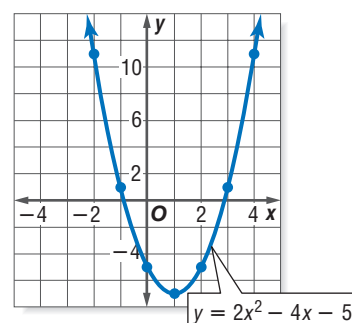


Example 1 Graph Opens Upward

Use a table of values to graph $y = 2x^2 - 4x - 5$.

Graph these ordered pairs and connect them with a smooth curve.

x	y
-2	11
-1	1
0	-5
1	-7
2	-5
3	1
4	11



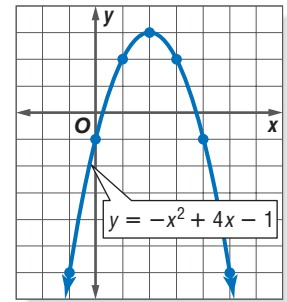
Consider the standard form $y = ax^2 + bx + c$. Notice that the value of a in Example 1 is positive and the curve opens upward. The lowest point, or **minimum**, of the graph is located at $(1, -7)$.

Example 2 Graph Opens Downward

Use a table of values to graph $y = -x^2 + 4x - 1$.

Graph these ordered pairs and connect them with a smooth curve.

x	y
-1	-6
0	-1
1	2
2	3
3	2
4	-1
5	-6



Study Tip

Reading Math

The plural of vertex is vertices. In math, vertex has several meanings. For example, there are the vertex of an angle, the vertices of a polygon, and the vertex of a parabola.

Notice that the value of a in Example 2 is negative and the curve opens downward. The highest point, or **maximum**, of the graph is located at $(2, 3)$. The maximum or minimum point of a parabola is called the **vertex**.

SYMMETRY AND VERTICES Parabolas possess a geometric property called **symmetry**. Symmetrical figures are those in which the figure can be folded in half so that each half matches the other exactly.



Algebra Activity

Symmetry of Parabolas

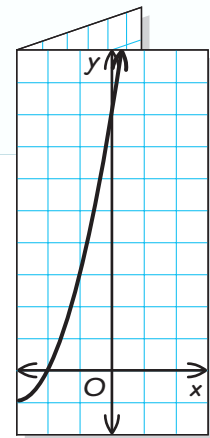


Model

- Graph $y = x^2 + 6x + 8$ on grid paper.
- Hold your paper up to the light and fold the parabola in half so that the two sides match exactly.
- Unfold the paper.

Make a Conjecture

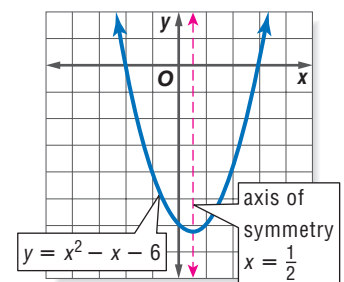
- What is the vertex of the parabola?
- Write an equation of the fold line.
- Which point on the parabola lies on the fold line?
- Write a few sentences to describe the symmetry of a parabola based on your findings in this activity.



The fold line in the activity above is called the **axis of symmetry** for the parabola. Each point on the parabola that is on one side of the axis of symmetry has a corresponding point on the parabola on the other side of the axis. The vertex is the only point on the parabola that is on the axis of symmetry.

In the graph of $y = x^2 - x - 6$, the axis of symmetry is $x = \frac{1}{2}$. The vertex is $(\frac{1}{2}, -6\frac{1}{4})$.

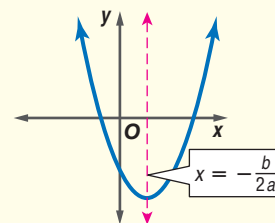
Notice the relationship between the values a and b and the equation of the axis of symmetry.



Key Concept Equation of the Axis of Symmetry of a Parabola

- **Words** The equation of the axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

- **Model**



You can determine information about a parabola from its equation.

Example 3 Vertex and Axis of Symmetry

Consider the graph of $y = -3x^2 - 6x + 4$.

- a. Write the equation of the axis of symmetry.

In $y = -3x^2 - 6x + 4$, $a = -3$ and $b = -6$.

$$x = -\frac{b}{2a} \quad \text{Equation for the axis of symmetry of a parabola}$$

$$x = -\frac{-6}{2(-3)} \text{ or } -1 \quad a = -3 \text{ and } b = -6$$

The equation of the axis of symmetry is $x = -1$.

- b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x -coordinate for the vertex is -1 .

$$y = -3x^2 - 6x + 4 \quad \text{Original equation}$$

$$y = -3(-1)^2 - 6(-1) + 4 \quad x = -1$$

$$y = -3 + 6 + 4 \quad \text{Simplify.}$$

$$y = 7 \quad \text{Add.}$$

The vertex is at $(-1, 7)$.

- c. Identify the vertex as a maximum or minimum.

Since the coefficient of the x^2 term is negative, the parabola opens downward and the vertex is a maximum point.

- d. Graph the function.

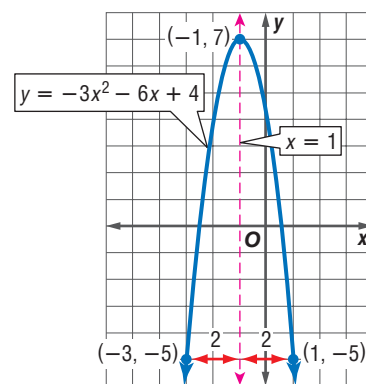
You can use the symmetry of the parabola to help you draw its graph. On a coordinate plane, graph the vertex and the axis of symmetry. Choose a value for x other than -1 . For example, choose 1 and find the y -coordinate that satisfies the equation.

$$y = -3x^2 - 6x + 4 \quad \text{Original equation}$$

$$y = -3(1)^2 - 6(1) + 4 \quad \text{Let } x = 1.$$

$$y = -5 \quad \text{Simplify.}$$

Graph $(1, -5)$. Since the graph is symmetrical about its axis of symmetry $x = -1$, you can find another point on the other side of the axis of symmetry. The point at $(1, -5)$ is 2 units to the right of the axis. Go 2 units to the left of the axis and plot the point $(-3, -5)$. Repeat this for several other points. Then sketch the parabola.



Study Tip

Coordinates of Vertex

Notice that you can find the x -coordinate by knowing the axis of symmetry. However, to find the y -coordinate, you must substitute the value of x into the quadratic equation.

CHECK Does $(-3, -5)$ satisfy the equation?

$$y = -3x^2 - 6x + 4 \quad \text{Original equation}$$

$$-5 \stackrel{?}{=} -3(-3)^2 - 6(-3) + 4 \quad y = -5 \text{ and } x = -3$$

$$-5 = -5 \checkmark \quad \text{Simplify.}$$

The ordered pair $(-3, -5)$ satisfies $y = -3x^2 - 6x + 4$, and the point is on the graph.

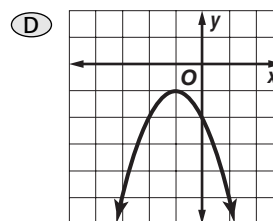
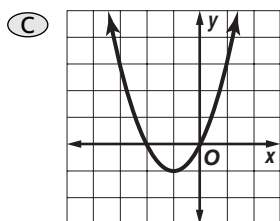
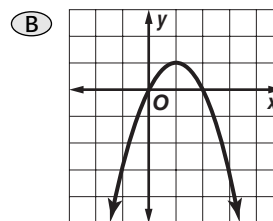
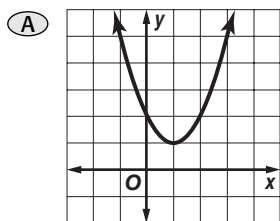
**Standardized
Test Practice**

A B C D

Example 4 Match Equations and Graphs

Multiple-Choice Test Item

Which is the graph of $y + 1 = (x + 1)^2$?



Read the Test Item

You are given a quadratic function, and you are asked to choose the graph that corresponds to it.

Solve the Test Item

First write the equation in standard form.

$$y + 1 = (x + 1)^2 \quad \text{Original equation}$$

$$y + 1 = x^2 + 2x + 1 \quad (x + 1)^2 = x^2 + 2x + 1$$

$$y + 1 - 1 = x^2 + 2x + 1 - 1 \quad \text{Subtract 1 from each side.}$$

$$y = x^2 + 2x \quad \text{Simplify.}$$

Then find the axis of symmetry of the graph of $y = x^2 + 2x$.

$$x = -\frac{b}{2a} \quad \text{Equation for the axis of symmetry}$$

$$x = -\frac{2}{2(1)} \text{ or } -1 \quad a = 1 \text{ and } b = 2$$

The axis of symmetry is $x = -1$. Look at the graphs. Since only choices C and D have this as their axis of symmetry, you can eliminate choices A and B. Since the coefficient of the x^2 term is positive, the graph opens upward. Eliminate choice D. The answer is C.



Test-Taking Tip

Sometimes you can answer a question by eliminating the incorrect choices. For example, in this test question, choices A and B are eliminated because their axes of symmetry are not $x = -1$.

Check for Understanding

Concept Check

1. Compare and contrast a parabola with a maximum and a parabola with a minimum.
2. **OPEN ENDED** Draw two different parabolas with a vertex of $(2, -1)$.
3. Explain how the axis of symmetry can help you graph a quadratic function.

Guided Practice

Use a table of values to graph each function.

4. $y = x^2 - 5$

5. $y = -x^2 + 4x + 5$

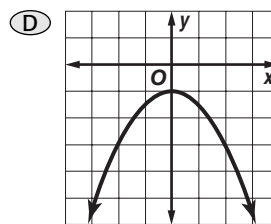
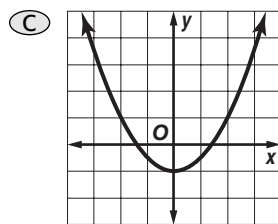
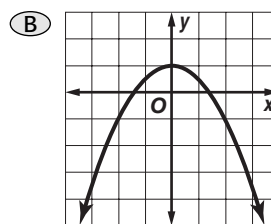
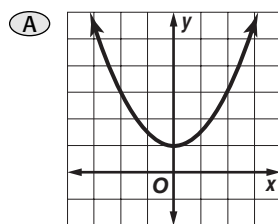
Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

6. $y = x^2 + 4x - 9$

7. $y = -x^2 + 5x + 6$

8. $y = -(x - 2)^2 + 1$

9. Which is the graph of $y = -\frac{1}{2}x^2 + 1$?



Standardized Test Practice

A B C D

Practice and Apply

Homework Help

For Exercises	See Examples
10–15	1, 2
16–49	3
52, 53	4

Extra Practice

See page 841.

Use a table of values to graph each function.

10. $y = x^2 - 3$

11. $y = -x^2 + 7$

12. $y = x^2 - 2x - 8$

13. $y = x^2 - 4x + 3$

14. $y = -3x^2 - 6x + 4$

15. $y = -3x^2 + 6x + 1$

16. What is the equation of the axis of symmetry of the graph of $y = -3x^2 + 2x - 5$?

17. Find the equation of the axis of symmetry of the graph of $y = 4x^2 - 5x + 16$.

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

18. $y = 4x^2$

19. $y = -2x^2$

20. $y = x^2 + 2$

21. $y = -x^2 + 5$

22. $y = -x^2 + 2x + 3$

23. $y = -x^2 - 6x + 15$

24. $y = x^2 - 14x + 13$

25. $y = x^2 + 2x + 18$

26. $y = 2x^2 + 12x - 11$

27. $y = 3x^2 - 6x + 4$

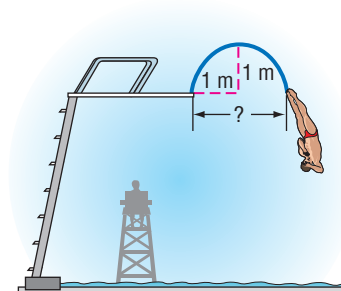
28. $y = 5 + 16x - 2x^2$

29. $y = 9 - 8x + 2x^2$

30. $y = 3(x + 1)^2 - 20$ 31. $y = -2(x - 4)^2 - 3$ 32. $y + 2 = x^2 - 10x + 25$
 33. $y + 1 = 3x^2 + 12x + 12$ 34. $y - 5 = \frac{1}{3}(x + 2)^2$ 35. $y + 1 = \frac{2}{3}(x + 1)^2$
36. The vertex of a parabola is at $(-4, -3)$. If one x -intercept is -11 , what is the other x -intercept?

37. What is the equation of the axis of symmetry of a parabola if its x -intercepts are -6 and 4 ?

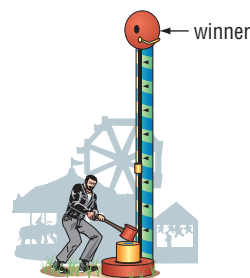
38. **SPORTS** A diver follows a path that is in the shape of a parabola. Suppose the diver's foot reaches 1 meter above the height of the diving board at the maximum height of the dive. At that time, the diver's foot is also 1 meter horizontally from the edge of the diving board. What is the distance of the diver's foot from the diving board as the diver descends past the diving board? Explain.



ENTERTAINMENT For Exercises 39 and 40, use the following information.

A carnival game involves striking a lever that forces a weight up a tube. If the weight reaches 20 feet to ring the bell, the contestant wins a prize. The equation $h = -16t^2 + 32t + 3$ gives the height of the weight if the initial velocity is 32 feet per second.

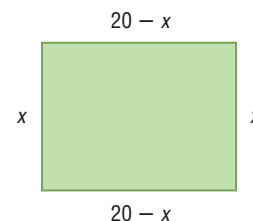
39. Find the maximum height of the weight.
 40. Will a prize be won?



PETS For Exercises 41–43, use the following information.

Miriam has 40 meters of fencing to build a pen for her dog.

41. Use the diagram at the right to write an equation for the area A of the pen.
 42. What value of x will result in the greatest area?
 43. What is the greatest possible area of the pen?



ARCHITECTURE For Exercises 44–46, use the following information.

The shape of the Gateway Arch in St. Louis, Missouri, is a *catenary curve*. It resembles a parabola with the equation $h = -0.00635x^2 + 4.0005x - 0.07875$, where h is the height in feet and x is the distance from one base in feet.

44. What is the equation of the axis of symmetry?
 45. What is the distance from one end of the arch to the other?
 46. What is the maximum height of the arch?

BRIDES For Exercises 47–49, use the following information.

The equation $a = 0.003x^2 - 0.115x + 21.3$ models the average ages of women when they first married since the year 1940. In this equation, a represents the average age and x represents the years since 1940.

47. Use what you know about parabolas and their minimum values to estimate the year in which the average age of brides was the youngest.
 48. Estimate the average age of the brides during that year.
 49. Use a graphing calculator to check your estimates.



More About . . .

Architecture

The Gateway Arch is part of a tribute to Thomas Jefferson, the Louisiana Purchase, and the pioneers who settled the West. Each year about 2.5 million people visit the arch.

Source: *World Book Encyclopedia*



50. **CRITICAL THINKING** Write a quadratic equation that represents a graph with an axis of symmetry with equation $x = -\frac{3}{8}$.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you coordinate a fireworks display with recorded music?

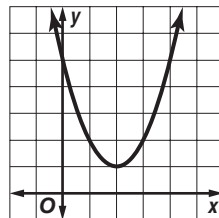
Include the following in your answer:

- an explanation of how to determine when the rocket will explode, and
- an explanation of how to determine the height of the rocket when it explodes.



52. Which equation corresponds to the graph at the right?

- (A) $y = x^2 - 4x + 5$
- (B) $y = -x^2 + 4x + 5$
- (C) $y = x^2 - 4x - 5$
- (D) $y = -x^2 + 4x - 5$



53. Which equation does *not* represent a quadratic function?

- (A) $y = (x + 3)^2$
- (B) $y = 3x^2$
- (C) $y = 6x^2 - 1$
- (D) $y = x + 5$



MAXIMUM OR MINIMUM Graph each function. Determine whether the vertex is a maximum or a minimum and give the ordered pair for the vertex.

- 54. $y = x^2 - 10x + 25$
- 55. $y = -x^2 + 4x + 3$
- 56. $y = -2x^2 - 8x - 1$
- 57. $y = 2x^2 - 40x + 214$
- 58. $y = 0.25x^2 - 4x - 2$
- 59. $y = -0.5x^2 - 2x + 3$

Maintain Your Skills

Mixed Review

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. (Lessons 9-5 and 9-6)

- 60. $x^2 + 6x - 9$
- 61. $a^2 + 22a + 121$
- 62. $4m^2 - 4m + 1$
- 63. $4q^2 - 9$
- 64. $2a^2 - 25$
- 65. $1 - 16g^2$

Find each sum or difference. (Lesson 8-5)

- 66. $(13x + 9y) + 11y$
- 67. $(7p^2 - p - 7) - (p^2 + 11)$

68. **RECREATION** At a recreation and sports facility, 3 members and 3 nonmembers pay a total of \$180 to take an aerobics class. A group of 5 members and 3 nonmembers pay \$210 to take the same class. How much does it cost members and nonmembers to take an aerobics class? (Lesson 7-3)

Solve each inequality. Then check your solution. (Lesson 6-2)

- 69. $12b > -144$
- 70. $-5w > -125$
- 71. $\frac{3r}{4} \leq \frac{2}{3}$

Write an equation of the line that passes through each point with the given slope. (Lesson 5-4)

- 72. $(2, 13), m = 4$
- 73. $(-2, -7), m = 0$
- 74. $(-4, 6), m = \frac{3}{2}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the x -intercept of the graph of each equation. (To review finding x -intercepts, see Lesson 4-5.)

- 75. $3x + 4y = 24$
- 76. $2x - 5y = 14$
- 77. $-2x - 4y = 7$
- 78. $7y + 6x = 42$
- 79. $2y - 4x = 10$
- 80. $3x - 7y + 9 = 0$





Graphing Calculator Investigation

A Follow-Up of Lesson 10-1

Families of Quadratic Graphs

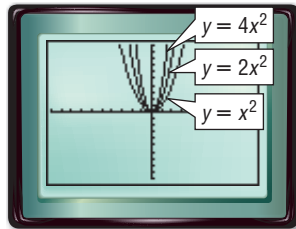
Recall that a *family of graphs* is a group of graphs that have at least one characteristic in common. On page 278, families of linear graphs were introduced. Families of quadratic graphs often fall into two categories—those that have the same vertex and those that have the same shape.

In each of the following families, the parent function is $y = x^2$. Graphing calculators make it easy to study the characteristics of these families of parabolas.

Graph each group of equations on the same screen. Use the standard viewing window. Compare and contrast the graphs.

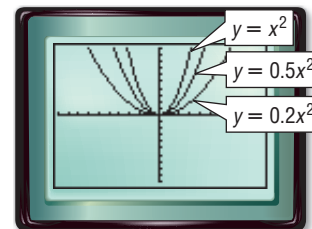
KEYSTROKES: Review graphing equations on pages 224 and 225.

a. $y = x^2$, $y = 2x^2$, $y = 4x^2$



Each graph opens upward and has its vertex at the origin. The graphs of $y = 2x^2$ and $y = 4x^2$ are narrower than the graph of $y = x^2$.

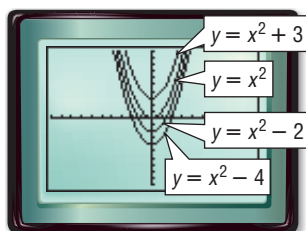
b. $y = x^2$, $y = 0.5x^2$, $y = 0.2x^2$



Each graph opens upward and has its vertex at the origin. The graphs of $y = 0.5x^2$ and $y = 0.2x^2$ are wider than the graph of $y = x^2$.

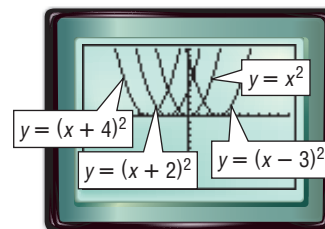
How does the value of a in $y = ax^2$ affect the shape of the graph?

c. $y = x^2$, $y = x^2 + 3$, $y = x^2 - 2$, $y = x^2 - 4$



Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex, located along the y -axis. How does the value of the constant affect the position of the graph?

d. $y = x^2$, $y = (x - 3)^2$, $y = (x + 2)^2$, $y = (x + 4)^2$



Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex located along the x -axis. How is the location of the vertex related to the equation of the graph?



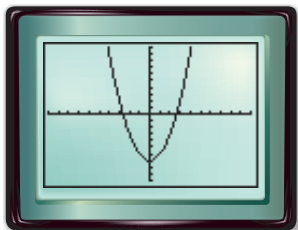
www.algebra1.com/other_calculator_keystrokes

Graphing Calculator Investigation

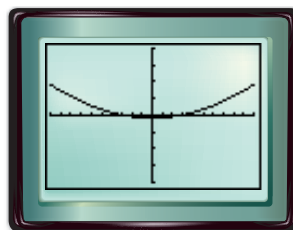
When analyzing or comparing the shapes of various graphs on different screens, it is important to compare the graphs using the same window with the same scale factors. Suppose you graph the same equation using a different window for each. How will the appearance of the graph change?

Graph $y = x^2 - 7$ in each viewing window. What conclusions can you draw about the appearance of a graph in the window used?

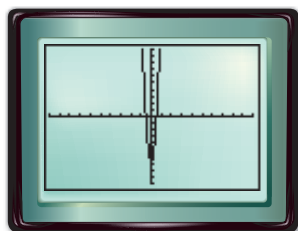
a. standard viewing window



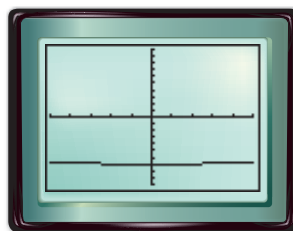
b. $[-10, 10]$ scl: 1 by $[-200, 200]$ scl: 50



c. $[-50, 50]$ scl: 5 by $[-10, 10]$ scl: 1



d. $[-0.5, 0.5]$ scl: 0.1 by $[-10, 10]$ scl: 1



The window greatly affects the appearance of the parabola. Without knowing the window, graph **b** might be of the family $y = ax^2$, where $0 < a < 1$. Graph **c** looks like a member of $y = ax^2 - 7$, where $a > 1$. Graph **d** looks more like a line. However, all are graphs of the same equation.

Exercises

Graph each family of equations on the same screen. Compare and contrast the graphs.

1. $y = -x^2$
 $y = -3x^2$
 $y = -6x^2$

2. $y = -x^2$
 $y = -0.6x^2$
 $y = -0.4x^2$

3. $y = -x^2$
 $y = -(x + 5)^2$
 $y = -(x - 4)^2$

4. $y = -x^2$
 $y = -x^2 + 7$
 $y = -x^2 - 5$

Use the families of graphs on page 531 and Exercises 1–4 above to predict the appearance of the graph of each equation. Then draw the graph.

5. $y = -0.1x^2$

6. $y = (x + 1)^2$

7. $y = 4x^2$

8. $y = x^2 - 6$

Describe how each change in $y = x^2$ would affect the graph of $y = x^2$. Be sure to consider all values of a , h , and k .

9. $y = ax^2$

10. $y = (x + h)^2$

11. $y = x^2 + k$

12. $y = (x + h)^2 + k$

Solving Quadratic Equations by Graphing

What You'll Learn

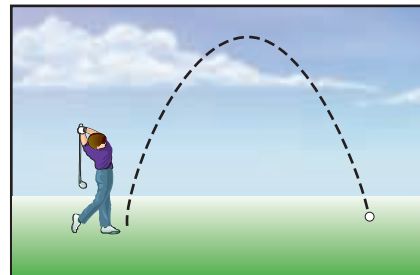
- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

Vocabulary

- quadratic equation
- roots
- zeros

How can quadratic equations be used in computer simulations?

A golf ball follows a path much like a parabola. Because of this property, quadratic functions can be used to simulate parts of a computer golf game. One of the x -intercepts of the quadratic function represents the location where the ball will hit the ground.



SOLVE BY GRAPHING Recall that a quadratic function has standard form $f(x) = ax^2 + bx + c$. In a **quadratic equation**, the value of the related quadratic function is 0. So for the quadratic equation $0 = x^2 - 2x - 3$, the related quadratic function is $f(x) = x^2 - 2x - 3$. You have used factoring to solve equations like $x^2 - 2x - 3 = 0$. You can also use graphing to determine the solutions of equations like this.

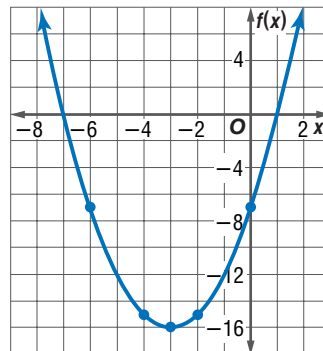
The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by finding the x -intercepts or **zeros** of the related quadratic function.

Example 1 Two Roots

Solve $x^2 + 6x - 7 = 0$ by graphing.

Graph the related function $f(x) = x^2 + 6x - 7$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or $x = -3$. When x equals -3 , $f(x)$ equals $(-3)^2 + 6(-3) - 7$ or -16 . So, the coordinates of the vertex are $(-3, -16)$. Make a table of values to find other points to sketch the graph.

x	$f(x)$
-8	9
-6	-7
-4	-15
-3	-16
-2	-15
0	-7
2	9



To solve $x^2 + 6x - 7 = 0$, you need to know where the value of $f(x)$ is 0. This occurs at the x -intercepts. The x -intercepts of the parabola appear to be -7 and 1 .

(continued on the next page)

CHECK Solve by factoring.

$$\begin{aligned}x^2 + 6x - 7 &= 0 && \text{Original equation} \\(x + 7)(x - 1) &= 0 && \text{Factor.} \\x + 7 = 0 &\quad \text{or} \quad x - 1 = 0 && \text{Zero Product Property} \\x = -7 &\quad \quad \quad x = 1 && \checkmark \text{ Solve for } x.\end{aligned}$$

The solutions of the equation are -7 and 1 .

Quadratic equations always have two roots. However, these roots are not always two distinct numbers. Sometimes the two roots are the same number.

Example 2 A Double Root

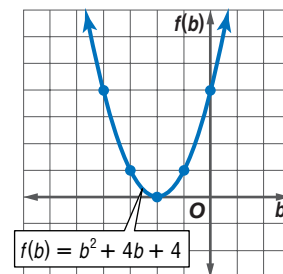
Solve $b^2 + 4b = -4$ by graphing.

First rewrite the equation so one side is equal to zero.

$$\begin{aligned}b^2 + 4b &= -4 && \text{Original equation} \\b^2 + 4b + 4 &= -4 + 4 && \text{Add 4 to each side.} \\b^2 + 4b + 4 &= 0 && \text{Simplify.}\end{aligned}$$

Graph the related function
 $f(b) = b^2 + 4b + 4$.

b	$f(b)$
-4	4
-3	1
-2	0
-1	1
0	4



Notice that the vertex of the parabola is the b -intercept. Thus, one solution is -2 . What is the other solution?

Try solving the equation by factoring.

$$\begin{aligned}b^2 + 4b + 4 &= 0 && \text{Original equation} \\(b + 2)(b + 2) &= 0 && \text{Factor.} \\b + 2 = 0 &\quad \text{or} \quad b + 2 = 0 && \text{Zero Product Property} \\b = -2 &\quad \quad \quad b = -2 && \text{Solve for } b.\end{aligned}$$

There are two identical factors for the quadratic function, so there is only one root, called a double root. The solution is -2 .

Study Tip

Common Misconception

Although solutions found by graphing may appear to be exact, you cannot be sure that they are exact. Solutions need to be verified by substituting into the equation and checking, or by using the algebraic methods that you will learn in this chapter.

Example 3 No Real Roots

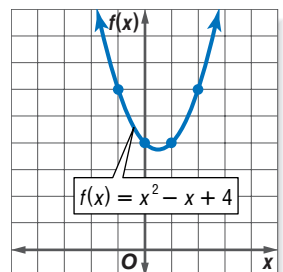
Solve $x^2 - x + 4 = 0$ by graphing.

Graph the related function
 $f(x) = x^2 - x + 4$.

The graph has no x -intercept. Thus, there are no real number solutions for this equation.

The symbol \emptyset , indicating an empty set, is often used to represent no real solutions.

x	$f(x)$
-1	6
0	4
1	4
2	6



ESTIMATE SOLUTIONS In Examples 1 and 2, the roots of the equation were integers. Usually the roots of a quadratic equation are not integers. In these cases, use estimation to approximate the roots of the equation.

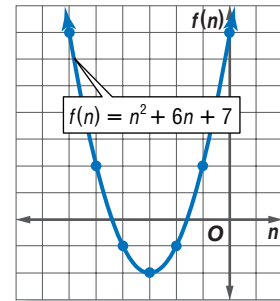
Example 4 Rational Roots

Solve $n^2 + 6n + 7 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

Graph the related function $f(n) = n^2 + 6n + 7$.

n	$f(n)$
-6	7
-5	2
-4	-1
-3	-2
-2	-1
-1	2
0	7

Notice that the value of the function changes from negative to positive between the n values of -5 and -4 and between -2 and -1 .

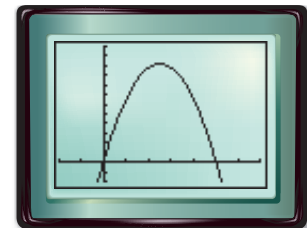


The n -intercepts of the graph are between -5 and -4 and between -2 and -1 . So, one root is between -5 and -4 , and the other root is between -2 and -1 .

Example 5 Estimate Solutions to Solve a Problem

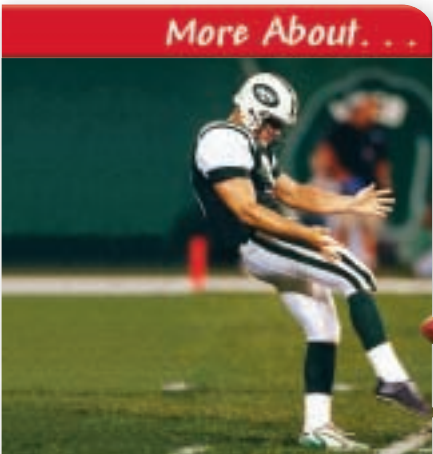
FOOTBALL When a football player punts a football, he hopes for a long “hang time.” Hang time is the total amount of time the ball stays in the air. A time longer than 4.5 seconds is considered good. If a punter kicks the ball with an upward velocity of 80 feet per second and his foot meets the ball 2 feet off the ground, the function $y = -16t^2 + 80t + 2$ represents the height of the ball y in feet after t seconds. What is the hang time of the ball?

You need to find the solution of the equation $0 = -16t^2 + 80t + 2$. Use a graphing calculator to graph the related function $y = -16t^2 + 80t + 2$. The x -intercept is about 5. Therefore, the hang time is about 5 seconds.



$[-2, 7]$ scl: 1 by $[-20, 120]$ scl: 10

Since 5 seconds is greater than 4.5 seconds, this kick would be considered to have good hang time.



More About . . .

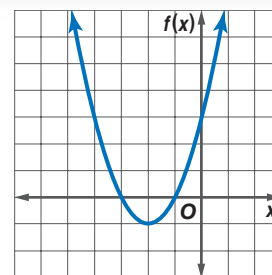
Football

On September 21, 1969, Steve O’Neal set a National Football League record by punting the ball 98 yards.

Source: *The Guinness Book of Records*

Check for Understanding

Concept Check 1. State the real roots of the quadratic equation whose related function is graphed at the right.



2. Write the related quadratic function for the equation $7x^2 + 2x = 8$.

3. **OPEN ENDED** Draw a graph to show a counterexample to the following statement.

All quadratic equations have two different solutions.

Guided Practice Solve each equation by graphing.

4. $x^2 - 7x + 6 = 0$ 5. $a^2 - 10a + 25 = 0$ 6. $c^2 + 3 = 0$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

7. $t^2 + 9t + 5 = 0$ 8. $x^2 - 16 = 0$ 9. $w^2 - 3w = 5$

- Application** 10. **NUMBER THEORY** Two numbers have a sum of 4 and a product of -12 . Use a quadratic equation to determine the two numbers.

Practice and Apply

Homework Help

For Exercises	See Examples
11–20	1–3
21–34	4
35–46	5

Extra Practice

See page 842.

Solve each equation by graphing.

11. $c^2 - 5c - 24 = 0$ 12. $5n^2 + 2n + 6 = 0$ 13. $x^2 + 6x + 9 = 0$
 14. $b^2 - 12b + 36 = 0$ 15. $x^2 + 2x + 5 = 0$ 16. $r^2 + 4r - 12 = 0$

17. The roots of a quadratic equation are -2 and -6 . The minimum point of the graph of its related function is at $(-4, -2)$. Sketch the graph of the function.
18. The roots of a quadratic equation are -6 and 0 . The maximum point of the graph of its related function is at $(-3, 4)$. Sketch the graph of the function.
19. **NUMBER THEORY** The sum of two numbers is 9 , and their product is 20 . Use a quadratic equation to determine the two numbers.
20. **NUMBER THEORY** Use a quadratic equation to find two numbers whose sum is 5 and whose product is -24 .

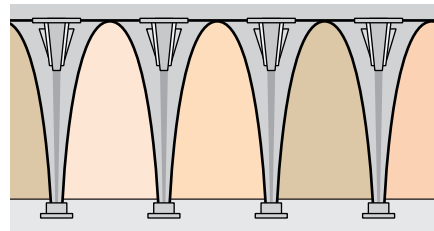
Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

21. $a^2 - 12 = 0$ 22. $n^2 - 7 = 0$ 23. $2c^2 + 20c + 32 = 0$
 24. $3s^2 + 9s - 12 = 0$ 25. $x^2 + 6x + 6 = 0$ 26. $y^2 - 4y + 1 = 0$
 27. $a^2 - 8a = 4$ 28. $x^2 + 6x = -7$ 29. $m^2 - 10m = -21$
 30. $p^2 + 16 = 8p$ 31. $12n^2 - 26n = 30$ 32. $4x^2 - 35 = -4x$

33. One root of a quadratic equation is between -4 and -3 , and the other root is between 1 and 2 . The maximum point of the graph of the related function is at $(-1, 6)$. Sketch the graph of the function.
34. One root of a quadratic equation is between -1 and 0 , and the other root is between 6 and 7 . The minimum point of the graph of the related function is at $(3, -5)$. Sketch the graph of the function.

- **DESIGN** For Exercises 35–39, use the following information.

An art gallery has walls that are sculptured with arches that can be represented by the quadratic function $f(x) = -x^2 - 4x + 12$, where x is in feet. The wall space under each arch is to be painted a different color from the arch itself.



35. Graph the quadratic function and determine its x -intercepts.
36. What is the length of the segment along the floor of each arch?

More About . . .



Design

The Winter Palace and the rest of the State Hermitage Museum in St. Petersburg, Russia, house 322 art galleries with about three million pieces of art.

Source: *The Guinness Book of Records*

37. What is the height of the arch?
38. The formula $A = \frac{2}{3}bh$ can be used to estimate the area under a parabola. In this formula, A represents area, b represents the length of the base, and h represents the height. Calculate the area that needs to be painted.
39. How much would the paint for the walls under 12 arches cost if the paint is \$27 per gallon, the painter applies 2 coats, and the manufacturer states that each gallon will cover 200 square feet? (*Hint*: Remember that you cannot buy part of a gallon.)

Web Quest

The graph of the surface areas of the planets can be modeled by a quadratic equation. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

40. **COMPUTER GAMES** Suppose the function $-0.005d^2 + 0.22d = h$ is used to simulate the path of a football at the kickoff of a computer football game. In this equation, h is the height of the ball and d is the horizontal distance in yards. What is the horizontal distance the ball will travel before it hits the ground?

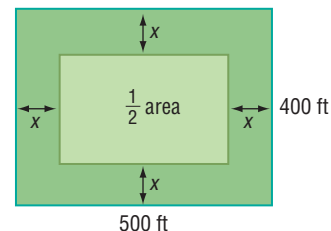
HIKING For Exercises 41 and 42, use the following information.

Monya and Kishi are hiking in the mountains and stop for lunch on a ledge 1000 feet above the valley below. Kishi decides to climb to another ledge 20 feet above Monya. Monya throws an apple up to Kishi, but Kishi misses it. The equation $h = -16t^2 + 30t + 1000$ represents the height in feet of the apple t seconds after it was thrown.

41. How long did it take for the apple to reach the ground?
42. If it takes 3 seconds to react, will the girls have time to call down and warn any hikers below? Assume that sound travels about 1000 feet per second. Explain.

WORK For Exercises 43–46, use the following information.

Kirk and Montega have accepted a job mowing the soccer playing fields. They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. They decide that Kirk will mow around the edge in a path of equal width until half the area is left.



43. What is the area each person will mow?
44. Write a quadratic equation that could be used to find the width x that Kirk should mow.
45. What width should Kirk mow?
46. The mower can mow a path 5 feet wide. To the nearest whole number, how many times should Kirk go around the field?
47. **CRITICAL THINKING** Where does the graph of $f(x) = \frac{x^3 + 2x^2 - 3x}{x + 5}$ intersect the x -axis?
48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

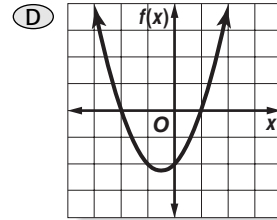
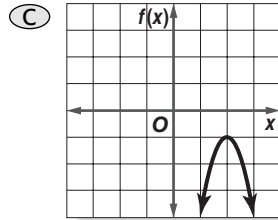
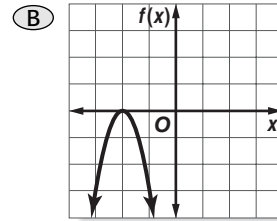
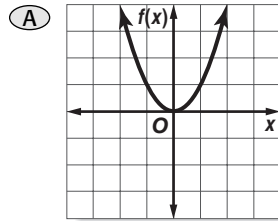
How can quadratic equations be used in computer simulations?

Include the following in your answer:

- the meaning of the two roots of a simulation equation for a computer golf game, and
- the approximate location at which the ball will hit the ground if the equation of the path of the ball is $y = -0.0015x^2 + 0.3x$, where y and x are in yards.

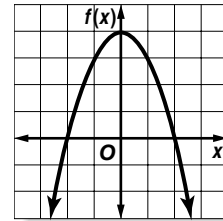


49. Which graph represents a function whose corresponding quadratic equation has no solutions?



50. What are the root(s) of the quadratic equation whose related function is graphed at the right?

- (A) -2, 2 (B) 0
(C) 4 (D) 0, 4



**Graphing
Calculator**

CUBIC EQUATIONS An equation of the form $ax^3 + bx^2 + cx + d = 0$ is called a **cubic equation**. You can use a graphing calculator to graph and solve cubic equations.

Use the graph of the related function of each cubic equation to estimate the roots of the equation.

51. $x^3 - x^2 - 4x + 4 = 0$

52. $2x^3 - 11x^2 + 13x - 4 = 0$

Maintain Your Skills

Mixed Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 10-1)

53. $y = x^2 + 6x + 9$

54. $y = -x^2 + 4x - 3$

55. $y = 0.5x^2 - 6x + 5$

Solve each equation. Check your solutions. (Lesson 9-6)

56. $m^2 - 24m = -144$

57. $7r^2 = 70r - 175$

58. $4d^2 + 9 = -12d$

Simplify. Assume that no denominator is equal to zero. (Lesson 8-2)

59. $\frac{10m^4}{30m}$

60. $\frac{22a^2b^5c^7}{-11abc^2}$

61. $\frac{-9m^3n^5}{27m^{-2}n^5y^{-4}}$

62. **SHIPPING** An empty book crate weighs 30 pounds. The weight of a book is 1.5 pounds. For shipping, the crate must weigh at least 55 pounds and no more than 60 pounds. What is the acceptable number of books that can be packed in the crate? (Lesson 6-4)

**Getting Ready for
the Next Lesson**

PREREQUISITE SKILL Determine whether each trinomial is a perfect square trinomial. If so, factor it. (To review **perfect square trinomials**, see Lesson 9-6.)

63. $a^2 + 14 + 49$

64. $m^2 - 10m + 25$

65. $t^2 + 16t - 64$

66. $4y^2 + 12y + 9$

67. $9d^2 - 12d - 4$

68. $25x^2 - 10x + 1$

Solving Quadratic Equations by Completing the Square

What You'll Learn

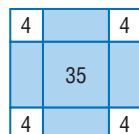
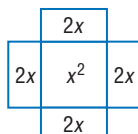
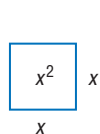
- Solve quadratic equations by finding the square root.
- Solve quadratic equations by completing the square.

Vocabulary

- completing the square

How did ancient mathematicians use squares to solve algebraic equations?

Al-Khwarizmi, born in Baghdad in 780, is considered to be one of the foremost mathematicians of all time. He wrote some of the oldest works on arithmetic and algebra. He wrote algebra in sentences instead of using equations, and he explained the work with geometric sketches. Al-Khwarizmi would have described $x^2 + 8x = 35$ as “A square and 8 roots are equal to 35 units.” He would solve the problem using the following sketch.



The area of the shaded portion is $x^2 + 8x$ or 35. Four squares each with an area of 4 are used to complete the square.

To solve problems this way today, you might use algebra tiles or a method called completing the square.

FIND THE SQUARE ROOT Some equations can be solved by taking the square root of each side.

Example 1 Irrational Roots

Solve $x^2 - 10x + 25 = 7$ by taking the square root of each side. Round to the nearest tenth if necessary.

$$x^2 - 10x + 25 = 7$$

Original equation

$$(x - 5)^2 = 7$$

$x^2 - 10x + 25$ is a perfect square trinomial.

$$\sqrt{(x - 5)^2} = \sqrt{7}$$

Take the square root of each side.

$$|x - 5| = \sqrt{7}$$

Simplify.

$$x - 5 = \pm\sqrt{7}$$

Definition of absolute value

$$x - 5 + 5 = \pm\sqrt{7} + 5$$

Add 5 to each side.

$$x = 5 \pm \sqrt{7}$$

Simplify.

Use a calculator to evaluate each value of x .

$$x = 5 + \sqrt{7} \quad \text{or} \quad x = 5 - \sqrt{7}$$

$$\approx 7.6 \quad \quad \quad \approx 2.4$$

The solution set is $\{2.4, 7.6\}$.

COMPLETE THE SQUARE To use the method shown in Example 1, the quadratic expression on one side of the equation must be a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, a method called **completing the square** may be used.

Study Tip

Look Back

To review **perfect square trinomials**, see Lesson 9-6.

Consider the pattern for squaring a binomial such as $x + 6$.

$$\begin{aligned}(x + 6)^2 &= x^2 + 2(6)(x) + 6^2 \\ &= x^2 + 12x + 36\end{aligned}$$

$$\begin{array}{ccc} \downarrow & & \uparrow \\ \left(\frac{12}{2}\right)^2 & \rightarrow & 6^2 \end{array}$$

Notice that one half of 12 is 6 and 6^2 is 36.

Key Concept

Completing the Square

To complete the square for a quadratic expression of the form $x^2 + bx$, you can follow the steps below.

Step 1 Find $\frac{1}{2}$ of b , the coefficient of x .

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$, the original expression.

Example 2 Complete the Square

Find the value of c that makes $x^2 + 6x + c$ a perfect square.

Method 1 Use algebra tiles.

Arrange the tiles for $x^2 + 6x$ so that the two sides of the figure are congruent.

To make the figure a square, add 9 positive 1-tiles.

$x^2 + 6x + 9$ is a perfect square.

Method 2 Complete the square.

Step 1 Find $\frac{1}{2}$ of 6. $\frac{6}{2} = 3$

Step 2 Square the result of Step 1. $3^2 = 9$

Step 3 Add the result of Step 2 to $x^2 + 6x$. $x^2 + 6x + 9$

Thus, $c = 9$. Notice that $x^2 + 6x + 9 = (x + 3)^2$.

Example 3 Solve an Equation by Completing the Square

Solve $a^2 - 14a + 3 = -10$ by completing the square.

Step 1 Isolate the a^2 and a terms.

$$a^2 - 14a + 3 = -10 \quad \text{Original equation}$$

$$a^2 - 14a + 3 - 3 = -10 - 3 \quad \text{Subtract 3 from each side.}$$

$$a^2 - 14a = -13 \quad \text{Simplify.}$$

Step 2 Complete the square and solve.

$$a^2 - 14a + 49 = -13 + 49 \quad \text{Since } \left(\frac{-14}{2}\right)^2 = 49, \text{ add 49 to each side.}$$

$$(a - 7)^2 = 36 \quad \text{Factor } a^2 - 14a + 49.$$

$$a - 7 = \pm 6 \quad \text{Take the square root of each side.}$$

$$a - 7 + 7 = \pm 6 + 7 \quad \text{Add 7 to each side.}$$

$$a = 7 \pm 6 \quad \text{Simplify.}$$

$$a = 7 + 6 \quad \text{or} \quad a = 7 - 6$$

$$= 13 \qquad \qquad = 1$$

CHECK Substitute each value for a in the original equation.

$$a^2 - 14a + 3 = -10 \qquad a^2 - 14a + 3 = -10$$

$$1^2 - 14(1) + 3 \stackrel{?}{=} -10 \qquad 13^2 - 14(13) + 3 \stackrel{?}{=} -10$$

$$1 - 14 + 3 \stackrel{?}{=} -10 \qquad 169 - 182 + 3 \stackrel{?}{=} -10$$

$$-10 = -10 \quad \checkmark \qquad -10 = -10 \quad \checkmark$$

The solution set is $\{1, 13\}$.

This method of completing the square cannot be used unless the coefficient of the first term is 1. To solve a quadratic equation in which the leading coefficient is not 1, first divide each term by the coefficient. Then follow the steps for completing the square.

Example 4 Solve a Quadratic Equation in Which $a \neq 1$

ENTERTAINMENT The path of debris from a firework display on a windy evening can be modeled by a quadratic function. A function for the path of the fireworks when the wind is about 15 miles per hour is $h = -0.04x^2 + 2x + 8$, where h is the height and x is the horizontal distance in feet. How far away from the launch site will the debris land?

Explore You know the function that relates the horizontal and vertical distances. You want to know how far away from the launch site the debris will land.

Plan The debris will hit the ground when $h = 0$. Use completing the square to solve $-0.04x^2 + 2x + 8 = 0$.

Solve	$-0.04x^2 + 2x + 8 = 0$	Equation for where debris will land
	$\frac{-0.04x^2 + 2x + 8}{-0.04} = \frac{0}{-0.04}$	Divide each side by -0.04 .
	$x^2 - 50x - 200 = 0$	Simplify.
	$x^2 - 50x - 200 + 200 = 0 + 200$	Add 200 to each side.
	$x^2 - 50x = 200$	Simplify.
	$x^2 - 50x + 625 = 200 + 625$	Since $(\frac{50}{2})^2 = 625$, add 625 to each side.
	$x^2 - 50x + 625 = 825$	Simplify.
	$(x - 25)^2 = 825$	Factor $x^2 - 50x + 625$.
	$x - 25 = \pm\sqrt{825}$	Take the square root of each side.
	$x - 25 + 25 = \pm\sqrt{825} + 25$	Add 25 to each side.
	$x = 25 \pm \sqrt{825}$	Simplify.

Use a calculator to evaluate each value of x .

$$x = 25 + \sqrt{825} \quad \text{or} \quad x = 25 - \sqrt{825}$$

$$\approx 53.7 \qquad \qquad \approx -3.7$$

Examine Since you are looking for a distance, ignore the negative number. The debris will land about 53.7 feet from the launch site.



More About . . .

Entertainment

One of the exploded fireworks for the Lake Toya Festival in Japan on July 15, 1988, broke a world record. The diameter of the burst was 3937 feet.

Source: *The Guinness Book of Records*



Check for Understanding

- Concept Check**
- OPEN ENDED** Make a square using one or more of the following types of tiles.
 - x^2 tile
 - x tile
 - 1 tile

Write an expression for the area of your square.

- Explain** why completing the square to solve $x^2 - 5x - 7 = 0$ is a better strategy than graphing the related function or factoring.
- Describe** the first step needed to solve $5x^2 + 12x = 15$ by completing the square.

Guided Practice Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

4. $b^2 - 6b + 9 = 25$

5. $m^2 + 14m + 49 = 20$

Find the value of c that makes each trinomial a perfect square.

6. $a^2 - 12a + c$

7. $t^2 + 5t + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

8. $c^2 - 6c = 7$

9. $x^2 + 7x = -12$

10. $v^2 + 14v - 9 = 6$

11. $r^2 - 4r = 2$

12. $a^2 - 24a + 9 = 0$

13. $2p^2 - 5p + 8 = 7$

- Application**
- GEOMETRY** The area of a square can be doubled by increasing the length by 6 inches and the width by 4 inches. What is the length of the side of the square?

Practice and Apply

Homework Help

For Exercises	See Examples
15–20	1
21–28	2
29–52	3, 4

Extra Practice

See page 842.

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

15. $b^2 - 4b + 4 = 16$

16. $t^2 + 2t + 1 = 25$

17. $g^2 - 8g + 16 = 2$

18. $y^2 - 12y + 36 = 5$

19. $w^2 + 16w + 64 = 18$

20. $a^2 + 18a + 81 = 90$

Find the value of c that makes each trinomial a perfect square.

21. $s^2 - 16s + c$

22. $y^2 - 10y + c$

23. $w^2 + 22w + c$

24. $a^2 + 34a + c$

25. $p^2 - 7p + c$

26. $k^2 + 11k + c$

27. Find all values of c that make $x^2 + cx + 81$ a perfect square.

28. Find all values of c that make $x^2 + cx + 144$ a perfect square.

Solve each equation by completing the square. Round to the nearest tenth if necessary.

29. $s^2 - 4s - 12 = 0$

30. $d^2 + 3d - 10 = 0$

31. $y^2 - 19y + 4 = 70$

32. $d^2 + 20d + 11 = 200$

33. $a^2 - 5a = -4$

34. $p^2 - 4p = 21$

35. $x^2 + 4x + 3 = 0$

36. $d^2 - 8d + 7 = 0$

37. $s^2 - 10s = 23$

38. $m^2 - 8m = 4$

39. $9r^2 + 49 = 42r$

40. $4h^2 + 25 = 20h$

41. $0.3t^2 + 0.1t = 0.2$

42. $0.4v^2 + 2.5 = 2v$

43. $5x^2 + 10x - 7 = 0$

44. $9w^2 - 12w - 1 = 0$

45. $\frac{1}{2}d^2 - \frac{5}{4}d - 3 = 0$

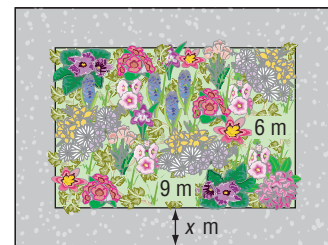
46. $\frac{1}{3}f^2 - \frac{7}{6}f + \frac{1}{2} = 0$

Solve each equation for x by completing the square.

47. $x^2 + 4x + c = 0$

48. $x^2 - 6x + c = 0$

49. **PARK PLANNING** A plan for a park has a rectangular plot of wild flowers that is 9 meters long by 6 meters wide. A pathway of constant width goes around the plot of wild flowers. If the area of the path is equal to the area of the plot of wild flowers, what is the width of the path?



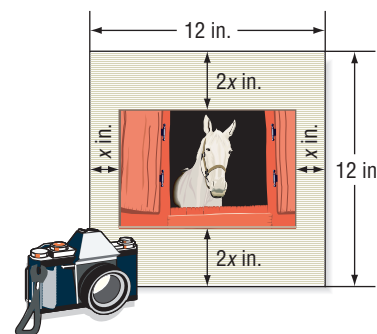
50. **EATING HABITS** In the early 1900s, the average American ate 300 pounds of bread and cereal every year. By the 1960s, Americans were eating half that amount. However, eating cereal and bread is on the rise again. The consumption of these types of foods can be modeled by the function $y = 0.059x^2 - 7.423x + 362.1$, where y represents the bread and cereal consumption in pounds and x represents the number of years since 1900. If this trend continues, in what future year will the average American consume 300 pounds of bread and cereal?



Online Research Data Update What are the eating habits of Americans? Visit www.algebra1.com/data_update to learn more.

51. **CRITICAL THINKING** Describe the solution of $x^2 + 4x + 12 = 0$. Explain your reasoning.

52. **PHOTOGRAPHY** Emilio is placing a photograph behind a 12-inch-by-12-inch piece of matting. The photograph is to be positioned so that the matting is twice as wide at the top and bottom as it is at the sides. If the area of the photograph is to be 54 square inches, what are the dimensions?



53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How did ancient mathematicians use squares to solve algebraic equations?

Include the following in your answer:

- an explanation of Al-Khwarizmi's drawings for $x^2 + 8x = 35$, and
- a step-by-step algebraic solution with justification for each step of the equation.

54. Determine which trinomial is *not* a perfect square trinomial.

(A) $a^2 - 26a + 169$

(B) $a^2 + 32a + 256$

(C) $a^2 + 30a - 225$

(D) $a^2 - 44a + 484$

55. Which equation is equivalent to $x^2 + 5x = 14$?

(A) $(x + \frac{5}{2})^2 = \frac{81}{4}$

(B) $(x - \frac{5}{2})^2 = \frac{45}{4}$

(C) $(x + \frac{5}{2})^2 = -\frac{5}{4}$

(D) $(x - \frac{5}{2})^2 = -\frac{5}{4}$

Career Choices

Photographer

Photographers must consider lighting, lens setting, and composition to create the best photograph.

Online Research

For information about a career as a photographer, visit: www.algebra1.com/careers



(A) (B) (C) (D)



Maintain Your Skills

Mixed Review Solve each equation by graphing. (Lesson 10-2)

56. $x^2 + 7x + 12 = 0$

57. $x^2 - 16 = 0$

58. $x^2 - 2x + 6 = 0$

Use a table of values to graph each equation. (Lesson 10-1)

59. $y = 4x^2 + 16$

60. $y = x^2 - 3x - 10$

61. $y = -x^2 + 3x - 4$

Find each GCF of the given monomials. (Lesson 9-1)

62. $14a^2b^3, 20a^3b^2c, 35ab^3c^2$

63. $32m^2n^3, 8m^2n, 56m^3n^2$

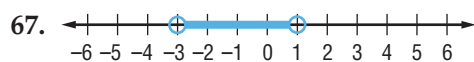
Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 7-2)

64. $y = 2x$
 $x + y = 9$

65. $x = y + 3$
 $2x - 3y = 5$

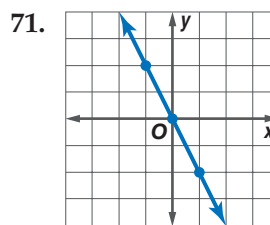
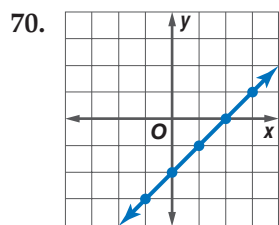
66. $x - 2y = 3$
 $3x + y = 23$

Write a compound inequality for each graph. (Lesson 6-4)



69. Write the slope-intercept form of an equation that passes through $(8, -2)$ and is perpendicular to the graph of $5x - 3y = 7$. (Lesson 5-6)

Write an equation for each relation. (Lesson 4-8)



Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary. (To review finding square roots, see Lesson 2-7.)

72. $a = 1, b = -2, c = -15$

73. $a = 2, b = 7, c = 3$

74. $a = 1, b = 5, c = -2$

75. $a = -2, b = 7, c = 5$

Practice Quiz 1

Lessons 10-1 through 10-3

Write the equation of the axis of symmetry and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 10-1)

1. $y = x^2 - x - 6$

2. $y = 2x^2 + 3$

3. $y = -3x^2 - 6x + 5$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. (Lesson 10-2)

4. $x^2 + 6x + 10 = 0$

5. $x^2 - 2x - 1 = 0$

6. $x^2 - 5x - 6 = 0$

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 10-3)

7. $s^2 + 8s = -15$

8. $a^2 - 10a = -24$

9. $y^2 - 14y + 49 = 5$

10. $2b^2 - b - 7 = 14$





Graphing Calculator Investigation

A Follow-Up of Lesson 10-3

Graphing Quadratic Functions in Vertex Form

Quadratic functions written in the form $y = a(x - h)^2 + k$ are said to be in **vertex form**.

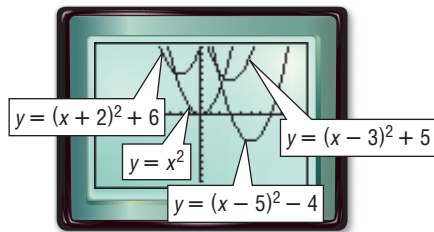
Graph each group of equations on the same screen. Use the standard viewing window. Compare and contrast the graphs.

a. $y = x^2$

$$y = (x - 3)^2 + 5$$

$$y = (x + 2)^2 + 6$$

$$y = (x - 5)^2 - 4$$



Each graph opens upward and has the same shape. However, the vertices are different.

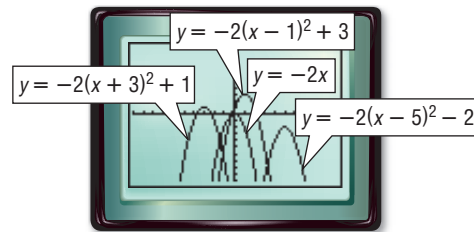
Equation	Vertex
$y = x^2$	(0, 0)
$y = (x - 3)^2 + 5$	(3, 5)
$y = (x + 2)^2 + 6$	(-2, 6)
$y = (x - 5)^2 - 4$	(5, -4)

b. $y = -2x^2$

$$y = -2(x - 1)^2 + 3$$

$$y = -2(x + 3)^2 + 1$$

$$y = -2(x - 5)^2 - 2$$



Each graph opens downward and has the same shape. However, the vertices are different.

Equation	Vertex
$y = -2x^2$	(0, 0)
$y = -2(x - 1)^2 + 3$	(1, 3)
$y = -2(x + 3)^2 + 1$	(-3, 1)
$y = -2(x - 5)^2 - 2$	(5, -2)

Exercises

- Study the relationship between the equations in vertex form and their vertices. What is the vertex of the graph of $y = a(x - h)^2 + k$?
- Completing the square can be used to change a quadratic equation to vertex form. Copy and complete the steps needed to rewrite $y = x^2 - 2x - 3$ in vertex form.

$$y = x^2 - 2x - 3$$

$$y = (x^2 - 2x + \underline{\quad? \quad}) - 3 - \underline{\quad? \quad}$$

$$y = (x - \underline{\quad? \quad})^2 - \underline{\quad? \quad}$$

Complete the square to rewrite each quadratic equation in vertex form. Then determine the vertex of the graph of the equation and sketch the graph.

3. $y = x^2 + 2x - 7$

4. $y = x^2 - 4x + 8$

5. $y = x^2 + 6x - 1$



www.algebra1.com/other_calculator_keystrokes

Solving Quadratic Equations by Using the Quadratic Formula

What You'll Learn

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number of solutions for a quadratic equation.

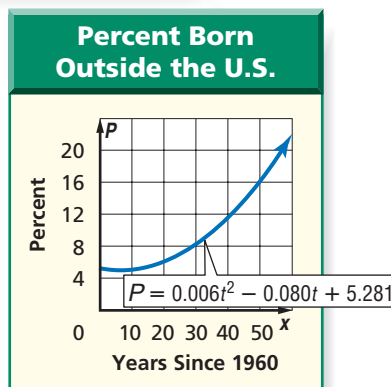
Vocabulary

- Quadratic Formula
- discriminant

How can the Quadratic Formula be used to solve problems involving population trends?

In the past few decades, there has been a dramatic increase in the percent of people living in the United States who were born in other countries. This trend can be modeled by the quadratic function $P = 0.006t^2 - 0.080t + 5.281$, where P is the percent born outside the United States and t is the number of years since 1960.

To predict when 15% of the population will be people who were born outside of the U.S., you can solve the equation $15 = 0.006t^2 - 0.080t + 5.281$. This equation would be impossible or difficult to solve using factoring, graphing, or completing the square.



QUADRATIC FORMULA You can solve the standard form of the quadratic equation $ax^2 + bx + c = 0$ for x . The result is called the **Quadratic Formula**.

Key Concept

The Quadratic Formula

The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can solve quadratic equations by factoring, graphing, completing the square, or using the Quadratic Formula.

Example 1 Integral Roots

Use two methods to solve $x^2 - 2x - 24 = 0$.

Method 1 Factoring

$$x^2 - 2x - 24 = 0 \quad \text{Original equation}$$

$$(x + 4)(x - 6) = 0 \quad \text{Factor } x^2 - 2x - 24.$$

$$x + 4 = 0 \quad \text{or} \quad x - 6 = 0 \quad \text{Zero Product Property}$$

$$x = -4 \quad \quad \quad x = 6 \quad \text{Solve for } x.$$

Study Tip

Look Back

To review **solving equations by factoring**, see Chapter 9.

Method 2 Quadratic Formula

For this equation, $a = 1$, $b = -2$, and $c = -24$.

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)} && a = 1, b = -2, \text{ and } c = -24 \\&= \frac{2 \pm \sqrt{4 + 96}}{2} && \text{Multiply.} \\&= \frac{2 \pm \sqrt{100}}{2} && \text{Add.} \\&= \frac{2 \pm 10}{2} && \text{Simplify.} \\x &= \frac{2 - 10}{2} \quad \text{or} \quad x = \frac{2 + 10}{2} \\&= -4 \qquad \qquad \qquad = 6\end{aligned}$$

The solution set is $\{-4, 6\}$.

Example 2 Irrational Roots

Solve $24x^2 - 14x = 6$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

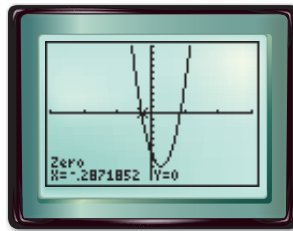
Step 1 Rewrite the equation in standard form.

$$\begin{aligned}24x^2 - 14x &= 6 && \text{Original equation} \\24x^2 - 14x - 6 &= 6 - 6 && \text{Subtract 6 from each side.} \\24x^2 - 14x - 6 &= 0 && \text{Simplify.}\end{aligned}$$

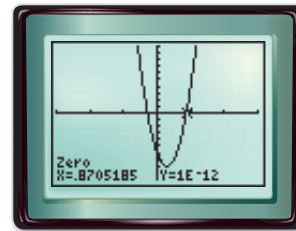
Step 2 Apply the Quadratic Formula.

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic Formula} \\&= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(24)(-6)}}{2(24)} && a = 24, b = -14, \text{ and } c = -6 \\&= \frac{14 \pm \sqrt{196 + 576}}{48} && \text{Multiply.} \\&= \frac{14 \pm \sqrt{772}}{48} && \text{Add.} \\x &= \frac{14 - \sqrt{772}}{48} \quad \text{or} \quad x = \frac{14 + \sqrt{772}}{48} \\&\approx -0.3 \qquad \qquad \qquad \approx 0.9\end{aligned}$$

Check the solutions by using the **CALC** menu on a graphing calculator to determine the zeros of the related quadratic function.



$[-3, 3]$ scl: 1 by $[-10, 10]$ scl: 1



$[-3, 3]$ scl: 1 by $[-10, 10]$ scl: 1

The approximate solution set is $\{-0.3, 0.9\}$.



You have studied four methods for solving quadratic equations. The table summarizes these methods.

Concept Summary		Solving Quadratic Equations
Method	Can Be Used	Comments
graphing	always	Not always exact; use only when an approximate solution is sufficient.
factoring	sometimes	Use if constant term is 0 or factors are easily determined.
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is an even number.
Quadratic Formula	always	Other methods may be easier to use in some cases, but this method always gives accurate solutions.

Example 3 Use the Quadratic Formula to Solve a Problem

SPACE TRAVEL The height H of an object t seconds after it is propelled upward with an initial velocity v is represented by $H = -\frac{1}{2}gt^2 + vt + h$, where g is the gravitational pull and h is the initial height. Suppose an astronaut on the Moon throws a baseball upward with an initial velocity of 10 meters per second, letting go of the ball 2 meters above the ground. Use the information at the left to find how much longer the ball will stay in the air than a similarly-thrown baseball on Earth.

In order to find when the ball hits the ground, you must find when $H = 0$. Write two equations to represent the situation on the Moon and on Earth.

Baseball Thrown on the Moon

$$\begin{aligned} H &= -\frac{1}{2}gt^2 + vt + h \\ 0 &= -\frac{1}{2}(1.6)t^2 + 10t + 2 \\ 0 &= -0.8t^2 + 10t + 2 \end{aligned}$$

Baseball Thrown on Earth

$$\begin{aligned} H &= -\frac{1}{2}gt^2 + vt + h \\ 0 &= -\frac{1}{2}(9.8)t^2 + 10t + 2 \\ 0 &= -4.9t^2 + 10t + 2 \end{aligned}$$

These equations cannot be factored, and completing the square would involve a lot of computation. To find accurate solutions, use the Quadratic Formula.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{10^2 - 4(-0.8)(2)}}{2(-0.8)} \\ &= \frac{-10 \pm \sqrt{106.4}}{-1.6} \\ t &\approx 12.7 \quad \text{or} \quad t \approx -0.2 \end{aligned}$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{10^2 - 4(-4.9)(2)}}{2(-4.9)} \\ &= \frac{-10 \pm \sqrt{139.2}}{-9.8} \\ t &\approx 2.2 \quad \text{or} \quad t \approx -0.2 \end{aligned}$$

Since a negative number of seconds is not reasonable, use the positive solutions. Therefore, the baseball will stay in the air about 12.7 seconds on the Moon and about 2.2 seconds on Earth. The baseball will stay in the air about $12.7 - 2.2$ or 10.5 seconds longer on the Moon.

More About . . .



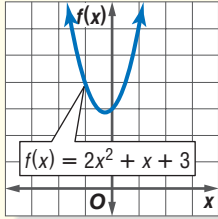
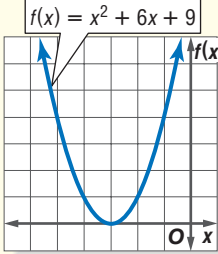
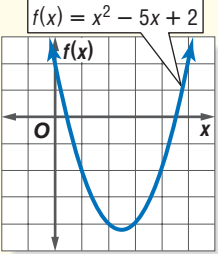
Space Travel

Astronauts have found walking on the Moon to be very different from walking on Earth because the gravitational pull of the moon is only 1.6 meters per second squared. The gravitational pull on Earth is 9.8 meters per second squared.

Source: World Book Encyclopedia

THE DISCRIMINANT In the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The value of the discriminant can be used to determine the number of real roots for a quadratic equation.



Discriminant	negative	zero	positive
Example	$2x^2 + x + 3 = 0$ $x = \frac{-1 \pm \sqrt{1^2 - 4(2)(3)}}{2(2)}$ $= \frac{-1 \pm \sqrt{-23}}{4}$ <p>There are no real roots since no real number can be the square root of a negative number.</p>	$x^2 + 6x + 9 = 0$ $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$ $= \frac{-6 \pm \sqrt{0}}{2}$ $= \frac{-6}{2} \text{ or } -3$ <p>There is a double root, -3.</p>	$x^2 - 5x + 2 = 0$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$ $= \frac{5 \pm \sqrt{17}}{2}$ <p>There are two roots, $\frac{5 + \sqrt{17}}{2}$ and $\frac{5 - \sqrt{17}}{2}$.</p>
Graph of Related Function	 <p>The graph does not cross the x-axis.</p>	 <p>The graph touches the x-axis in one place.</p>	 <p>The graph crosses the x-axis twice.</p>
Number of Real Roots	0	1	2

Example 4 Use the Discriminant

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

a. $2x^2 + 10x + 11 = 0$

$$b^2 - 4ac = 10^2 - 4(2)(11) \quad a = 2, b = 10, \text{ and } c = 11$$

$$= 12 \quad \text{Simplify.}$$

Since the discriminant is positive, the equation has two real roots.

b. $4t^2 - 20t + 25 = 0$

$$b^2 - 4ac = (-20)^2 - 4(4)(25) \quad a = 4, b = -20, \text{ and } c = 25$$

$$= 0 \quad \text{Simplify.}$$

Since the discriminant is 0, the equation has one real root.

c. $3m^2 + 4m = -2$

Step 1 Rewrite the equation in standard form.

$$3m^2 + 4m = -2 \quad \text{Original equation}$$

$$3m^2 + 4m + 2 = -2 + 2 \quad \text{Add 2 to each side.}$$

$$3m^2 + 4m + 2 = 0 \quad \text{Simplify.}$$

Step 2 Find the discriminant.

$$b^2 - 4ac = 4^2 - 4(3)(2) \quad a = 3, b = 4, \text{ and } c = 2$$

$$= -8 \quad \text{Simplify.}$$

Since the discriminant is negative, the equation has no real roots.

Check for Understanding

Concept Check

1. **Describe** three different ways to solve $x^2 - 2x - 15 = 0$. Which method do you prefer and why?
2. **OPEN ENDED** Write a quadratic equation with no real solutions.
3. **FIND THE ERROR** Lakeisha and Juanita are determining the number of solutions of $5y^2 - 3y = 2$.

Lakeisha

$$5y^2 - 3y = 2$$

$$b^2 - 4ac = (-3)^2 - 4(5)(2)$$

$$= -31$$

Since the discriminant is negative, there are no real solutions.

Juanita

$$5y^2 - 3y = 2$$

$$5y^2 - 3y - 2 = 0$$

$$b^2 - 4ac = (-3)^2 - 4(5)(-2)$$

$$= 49$$

Since the discriminant is positive, there are two real roots.

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

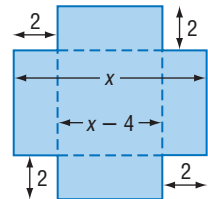
4. $x^2 + 7x + 6 = 0$
5. $t^2 + 11t = 12$
6. $r^2 + 10r + 12 = 0$
7. $3v^2 + 5v + 11 = 0$
8. $4x^2 + 2x = 17$
9. $w^2 + \frac{2}{25} = \frac{3}{5}w$

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

10. $m^2 + 5m - 6 = 0$
11. $s^2 + 8s + 16 = 0$
12. $2z^2 + z = -50$

Application

13. **MANUFACTURING** A pan is to be formed by cutting 2-centimeter-by-2-centimeter squares from each corner of a square piece of sheet metal and then folding the sides. If the volume of the pan is to be 441 square centimeters, what should the dimensions of the original piece of sheet metal be?



Practice and Apply

Homework Help

For Exercises	See Examples
14–37	1, 2
38–45	4
46–53	3

Extra Practice

See page 842.

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

14. $x^2 + 3x - 18 = 0$
15. $v^2 + 12v + 20 = 0$
16. $3t^2 - 7t - 20 = 0$
17. $5y^2 - y - 4 = 0$
18. $x^2 - 25 = 0$
19. $r^2 + 25 = 0$
20. $2x^2 + 98 = 28x$
21. $4s^2 + 100 = 40s$
22. $2r^2 + r - 14 = 0$
23. $2n^2 - 7n - 3 = 0$
24. $5v^2 - 7v = 1$
25. $11z^2 - z = 3$
26. $2w^2 = -(7w + 3)$
27. $2(12g^2 - g) = 15$
28. $1.34d^2 - 1.1d = -1.02$
29. $-2x^2 + 0.7x = -0.3$
30. $2y^2 - \frac{5}{4}y = \frac{1}{2}$
31. $\frac{1}{2}v^2 - v = \frac{3}{4}$

32. **GEOMETRY** The perimeter of a rectangle is 60 inches. Find the dimensions of the rectangle if its area is 221 square inches.

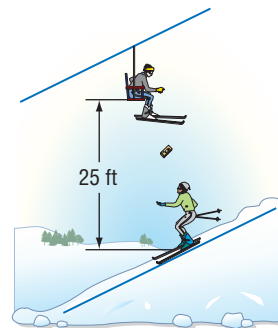
33. **GEOMETRY** Rectangle $ABCD$ has a perimeter of 42 centimeters. What are the dimensions of the rectangle if its area is 80 square centimeters?
34. **NUMBER THEORY** Find two consecutive odd integers whose product is 255.
35. **NUMBER THEORY** The sum of the squares of two consecutive odd numbers is 130. What are the numbers?
36. Without graphing, determine the x -intercepts of the graph of $f(x) = 4x^2 - 9x + 4$.
37. Without graphing, determine the x -intercepts of the graph of $f(x) = 13x^2 - 16x - 4$.

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

38. $x^2 + 3x - 4 = 0$ 39. $y^2 + 3y + 1 = 0$ 40. $4p^2 + 10p = -6.25$
41. $1.5m^2 + m = -3.5$ 42. $2r^2 = \frac{1}{2}r - \frac{2}{3}$ 43. $\frac{4}{3}n^2 + 4n = -3$
44. Without graphing, determine the number of x -intercepts of the graph of $f(x) = 7x^2 - 3x - 1$.
45. Without graphing, determine the number of x -intercepts of the graph of $f(x) = x^2 + 4x + 7$.

• **RECREATION** For Exercises 46 and 47, use the following information.

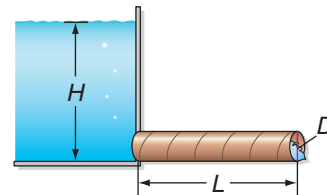
As Darius is skiing down a ski slope, Jorge is on the chair lift on the same slope. The chair lift has stopped. Darius stops directly below Jorge and attempts to toss a disposable camera up to him. If the camera is thrown with an initial velocity of 35 feet per second, the equation for the height of the camera is $h = -16t^2 + 35t + 5$, where h represents the height in feet and t represents the time in seconds.



46. If the chair lift is 25 feet above the ground, will Jorge have 0, 1, or 2 chances to catch the camera?
47. If Jorge is unable to catch the camera, when will it hit the ground?

48. **PHYSICAL SCIENCE** A projectile is shot vertically up in the air from ground level. Its distance s , in feet, after t seconds is given by $s = 96t - 16t^2$. Find the values of t when s is 96 feet.

49. **WATER MANAGEMENT** Cox's formula for measuring velocity of water draining from a reservoir through a horizontal pipe is $4v^2 + 5v - 2 = \frac{1200HD}{L}$, where v represents the velocity of the water in feet per second, H represents the height of the reservoir in feet, D represents the diameter of the pipe in inches, and L represents the length of the pipe in feet. How fast is water flowing through a pipe 20 feet long with a diameter of 6 inches that is draining a swimming pool with a depth of 10 feet?



50. **CRITICAL THINKING** If the graph of $f(x) = ax^2 + 10x + 3$ intersects the x -axis in two places, what must be true about the value of a ?



Recreation

Downhill skiing is the most popular type of snow skiing. Skilled skiers can obtain speeds of about 60 miles per hour as they race down mountain slopes.

Source: *World Book Encyclopedia*





Graphing Calculator Investigation

A Follow-Up of Lesson 10-4

Solving Quadratic-Linear Systems

Since you can graph multiple functions on a graphing calculator, it is a useful tool when finding the intersection points or solutions of a system of equations in which one equation is quadratic and one is linear.

Solve the following quadratic-linear system of equations.

$$y + 1 = x$$
$$y = -x^2 + 2x + 5$$

Step 1 Solve each equation for y .

- $y + 1 = x$
 $y = x - 1$
- $y = -x^2 + 2x + 5$

Step 2 Graph the equations on the same screen.

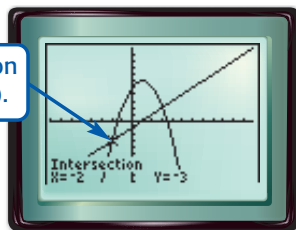
- Enter $y = x - 1$ as Y_1 .
- Enter $y = -x^2 + 2x + 5$ as Y_2 .
- Graph both in the standard viewing window.

Step 3 Approximate the intersection point.

- Use the intersect option on the CALC menu to approximate the first intersection point.

KEYSTROKES: **2nd** [CALC] 5 **ENTER** **ENTER**
ENTER

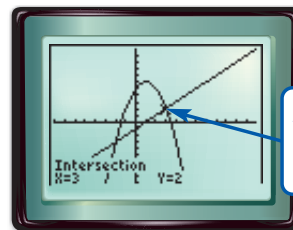
One solution is $(-2, -3)$.



Step 4 Approximate the other intersection point.

- Use the TRACE feature with the right and left arrow keys to move the cursor near the other intersection point.
- Use the intersect option on the CALC menu to approximate the other intersection point.

A second solution is $(3, 2)$.



Thus, the solutions of the quadratic-linear system are $(-2, -3)$ and $(3, 2)$.

Exercises

Use the intersect feature to solve each quadratic-linear system of equations. State any decimal solutions to the nearest tenth.

1. $y = -2(2x + 3)$
 $y = x^2 + 2x + 3$

2. $y - 5 = 0$
 $y = -x^2$

3. $1.8x + y = 3.6$
 $y = x^2 - 3x - 1$

4. $y = -1.4x - 2.88$
 $y = x^2 + 0.4x - 3.14$

5. $y = x^2 - 3.5x + 2.2$
 $y = 2x - 5.3625$

6. $y = 0.35x - 1.648$
 $y = -0.2x^2 + 0.28x + 1.01$



www.algebra1.com/other_calculator_keystrokes

What You'll Learn

- Graph exponential functions.
- Identify data that displays exponential behavior.

Vocabulary

- exponential function

How can exponential functions be used in art?

Earnest “Mooney” Warther was a whittler and a carver. For one of his most unusual carvings, Mooney carved a large pair of pliers in a tree.

From this original carving, he carved another pair of pliers in each handle of the original. Then he carved another pair of pliers in each of those handles. He continued this pattern to create the original pliers and 8 more layers of pliers. Even more amazing is the fact that all of the pliers work.



The number of pliers on each level is given in the table below.

Level	Number of Pliers	Power of 2
Original	1	2^0
First	$1(2) = 2$	2^1
Second	$2(2) = 4$	2^2
Third	$2(2)(2) = 8$	2^3
Fourth	$2(2)(2)(2) = 16$	2^4
Fifth	$2(2)(2)(2)(2) = 32$	2^5
Sixth	$2(2)(2)(2)(2)(2) = 64$	2^6
Seventh	$2(2)(2)(2)(2)(2)(2) = 128$	2^7
Eighth	$2(2)(2)(2)(2)(2)(2)(2) = 256$	2^8

GRAPH EXPONENTIAL FUNCTIONS Study the Power of 2 column. Notice that the exponent number matches the level. So we can write an equation to describe y , the number of pliers for any given level x as $y = 2^x$. This type of function, in which the variable is the exponent, is called an **exponential function**.

Key Concept**Exponential Function**

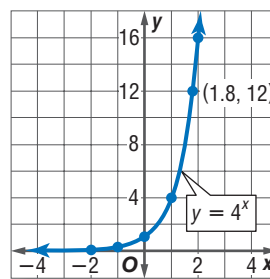
An exponential function is a function that can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

As with other functions, you can use ordered pairs to graph an exponential function.

Example 1 Graph an Exponential Function with $a > 1$

- a. Graph $y = 4^x$. State the y -intercept.

x	4^x	y
-2	4^{-2}	$\frac{1}{16}$
-1	4^{-1}	$\frac{1}{4}$
0	4^0	1
1	4^1	4
2	4^2	16
3	4^3	64



Graph the ordered pairs and connect the points with a smooth curve. The y -intercept is 1. Notice that the y values change little for small values of x , but they increase quickly as the values of x become greater.

- b. Use the graph to determine the approximate value of $4^{1.8}$.

The graph represents all real values of x and their corresponding values of y for $y = 4^x$. So, the value of y is about 12 when $x = 1.8$. Use a calculator to confirm this value.

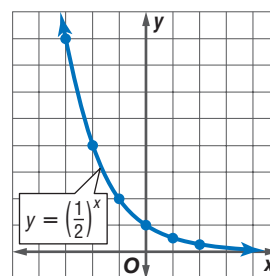
$$4^{1.8} \approx 12.12573252$$

The graphs of functions of the form $y = a^x$, where $a > 1$, all have the same shape as the graph in Example 1, rising faster and faster as you move from left to right.

Example 2 Graph Exponential Functions with $0 < a < 1$

- a. Graph $y = \left(\frac{1}{2}\right)^x$. State the y -intercept.

x	$\left(\frac{1}{2}\right)^x$	y
-3	$\left(\frac{1}{2}\right)^{-3}$	8
-2	$\left(\frac{1}{2}\right)^{-2}$	4
-1	$\left(\frac{1}{2}\right)^{-1}$	2
0	$\left(\frac{1}{2}\right)^0$	1
1	$\left(\frac{1}{2}\right)^1$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2$	$\frac{1}{4}$



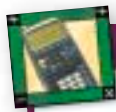
Graph the ordered pairs and connect the points with a smooth curve. The y -intercept is 1. Notice that the y values decrease less rapidly as x increases.

- b. Use the graph to determine the approximate value of $\left(\frac{1}{2}\right)^{-2.5}$.

The value of y is about $5\frac{1}{2}$ when $x = -2.5$. Use a calculator to confirm this value.

$$\left(\frac{1}{2}\right)^{-2.5} \approx 5.656854249$$

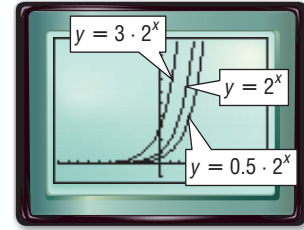




Graphing Calculator Investigation

Transformations of Exponential Functions

You can use a graphing calculator to study families of graphs of exponential functions. For example, the graph at the right shows the graphs of $y = 2^x$, $y = 3 \cdot 2^x$, and $y = 0.5 \cdot 2^x$. Notice that the y -intercept of $y = 2^x$ is 1, the y -intercept of $y = 3 \cdot 2^x$ is 3, and the y -intercept of $y = 0.5 \cdot 2^x$ is 0.5. The graph of $y = 3 \cdot 2^x$ is steeper than the graph of $y = 2^x$. The graph of $y = 0.5 \cdot 2^x$ is not as steep as the graph of $y = 2^x$.



$[-10, 10]$ scl: 1 by $[-1, 10]$ scl: 1

Think and Discuss

Graph each family of equations on the same screen. Compare and contrast the graphs.

1. $y = 2^x$
 $y = 2^x + 3$
 $y = 2^x - 4$

2. $y = 2^x$
 $y = 2^{x+5}$
 $y = 2^{x-4}$

3. $y = 2^x$
 $y = 3^x$
 $y = 5^x$

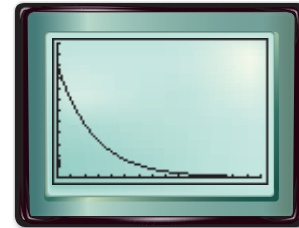
4. $y = 3 \cdot 2^x$
 $y = 3(2^x - 1)$
 $y = 3(2^x + 1)$

Example 3 Use Exponential Functions to Solve Problems

MOTION PICTURES Movies tend to have their best ticket sales the first weekend after their release. The sales then follow a decreasing exponential function each successive weekend after the opening. The function $E = 49.9 \cdot 0.692^w$ models the earnings of a popular movie. In this equation, E represents earnings in millions of dollars and w represents the weekend number.

a. Graph the function. What values of E and w are meaningful in the context of the problem?

Use a graphing calculator to graph the function. Only values where $E \leq 49.9$ and $w > 0$ are meaningful in the context of the problem.



$[0, 15]$ scl: 1 by $[0, 60]$ scl: 5

b. How much did the movie make on the first weekend?

$E = 49.9 \cdot 0.692^w$ Original equation

$E = 49.9 \cdot 0.692^1$ $w = 1$

$E = 34.5308$ Use a calculator.

On the first weekend, the movie grossed about \$34.53 million.

c. How much did it make on the fifth weekend?

$E = 49.9 \cdot 0.692^w$ Original equation

$E = 49.9 \cdot 0.692^5$ $w = 5$

$E \approx 7.918282973$ Use a calculator.

On the fifth weekend, the movie grossed about \$7.92 million.

More About...



Motion Pictures

The first successful photographs of motion were made in 1877. Today, the motion picture industry is big business, with the highest-grossing movie making \$600,800,000.

Source: World Book Encyclopedia



IDENTIFY EXPONENTIAL BEHAVIOR How do you know if a set of data is exponential? One method is to observe the shape of the graph. But the graph of an exponential function may resemble part of the graph of a quadratic function. Another way is to use the problem-solving strategy *look for a pattern* with the data.

Example 4 Identify Exponential Behavior

Determine whether each set of data displays exponential behavior.

a.

x	0	10	20	30	40	50
y	80	40	20	10	5	2.5

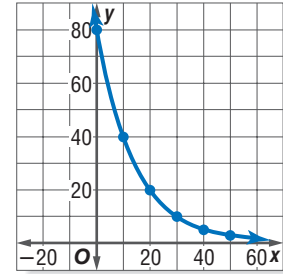
Method 1 Look for a Pattern

The domain values are at regular intervals of 10. Let's see if there is a common factor among the range values.

$$\begin{array}{cccccc}
 80 & 40 & 20 & 10 & 5 & 2.5 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 \times \frac{1}{2} & \times \frac{1}{2} & \times \frac{1}{2} & \times \frac{1}{2} & \times \frac{1}{2} &
 \end{array}$$

Since the domain values are at regular intervals and the range values have a common factor, the data are probably exponential. The equation for the data may involve $\left(\frac{1}{2}\right)^x$.

Method 2 Graph the Data



The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior.

b.

x	0	10	20	30	40	50
y	15	21	27	33	39	45

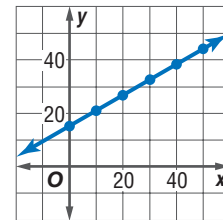
Method 1 Look for a Pattern

The domain values are at regular intervals of 10. The range values have a common difference 6.

$$\begin{array}{cccccc}
 15 & 21 & 27 & 33 & 39 & 45 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 + 6 & + 6 & + 6 & + 6 & + 6 &
 \end{array}$$

The data do not display exponential behavior, but rather linear behavior.

Method 2 Graph the Data



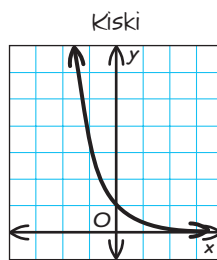
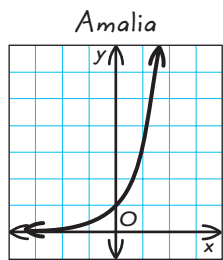
This is a graph of a line, not an exponential function.

Check for Understanding

- Concept Check**
- Determine** whether the graph of $y = a^x$, where $a > 0$ and $a \neq 1$, sometimes, always, or never has an x -intercept.
 - OPEN ENDED** Write an exponential function and graph the function. Describe the graph.



3. **FIND THE ERROR** Amalia and Kiski are graphing $y = \left(\frac{1}{3}\right)^x$.



Who is correct? Explain your reasoning.

Guided Practice

Graph each function. State the y -intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

4. $y = 3^x$; $3^{1.2}$ 5. $y = \left(\frac{1}{4}\right)^x$; $\left(\frac{1}{4}\right)^{1.7}$ 6. $y = 9^x$; $9^{0.8}$

Graph each function. State the y -intercept.

7. $y = 2 \cdot 3^x$ 8. $y = 4(5^x - 10)$

Determine whether the data in each table display exponential behavior. Explain why or why not.

9.

x	0	1	2	3	4	5
y	1	6	36	216	1296	7776

10.

x	4	6	8	10	12	14
y	5	9	13	17	21	25

Application

FOLKLORE For Exercises 11 and 12, use the following information.

A wise man asked his ruler to provide rice for feeding his people. Rather than receiving a constant daily supply of rice, the wise man asked the ruler to give him 2 grains of rice for the first square on a chessboard, 4 grains for the second, 8 grains for the third, 16 for the fourth, and so on doubling the amount of rice with each square of the board.

11. How many grains of rice will the wise man receive for the last (64th) square on the chessboard?
12. If one pound of rice has approximately 24,000 grains, how many tons of rice will the wise man receive on the last day? (*Hint*: one ton = 2000 pounds)

Practice and Apply

Homework Help

For Exercises	See Examples
13–26	1, 2
27–32	4
33–41	3

Graph each function. State the y -intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

13. $y = 5^x$; $5^{1.1}$ 14. $y = 10^x$; $10^{0.3}$ 15. $y = \left(\frac{1}{10}\right)^x$; $\left(\frac{1}{10}\right)^{-1.3}$
16. $y = \left(\frac{1}{5}\right)^x$; $\left(\frac{1}{5}\right)^{0.5}$ 17. $y = 6^x$; $6^{0.3}$ 18. $y = 8^x$; $8^{0.8}$

Extra Practice

See page 843.

Graph each function. State the y -intercept.

19. $y = 5(2^x)$ 20. $y = 3(5^x)$ 21. $y = 3^x - 7$ 22. $y = 2^x + 4$
23. $y = 2(3^x) - 1$ 24. $y = 5(2^x) + 4$ 25. $y = 2(3^x + 1)$ 26. $y = 3(2^x - 5)$



Determine whether the data in each table display exponential behavior. Explain why or why not.

27.

x	-2	-1	0	1
y	-5	-2	1	4

28.

x	0	1	2	3
y	1	0.5	0.25	0.125

29.

x	10	20	30	40
y	16	12	9	6.75

30.

x	-1	0	1	2
y	-0.5	1.0	-2.0	4.0

31.

x	3	6	9	12
y	5	5	5	5

32.

x	5	3	1	-1
y	32	16	8	4

BUSINESS For Exercises 33–35, use the following information.

The amount of money spent at West Outlet Mall in Midtown continues to increase. The total $T(x)$ in millions of dollars can be estimated by the function $T(x) = 12(1.12)^x$, where x is the number of years after it opened in 1995.

33. According to the function, find the amount of sales for the mall in the years 2005, 2006, and 2007.
 34. Graph the function and name the y -intercept.
 35. What does the y -intercept represent in this problem?

36. **BIOLOGY** Mitosis is a process of cell reproduction in which one cell divides into two identical cells. *E. coli* is a fast-growing bacterium that is often responsible for food poisoning in uncooked meat. It can reproduce itself in 15 minutes. If you begin with 100 *E. coli* bacteria, how many will there be in one hour?

TOURNAMENTS For Exercises 37–39, use the following information.

In a regional quiz bowl competition, three schools compete and the winner advances to the next round. Therefore, after each round, only $\frac{1}{3}$ of the schools remain in the competition for the next round. Suppose 729 schools start the competition.

37. Write an exponential function to describe the number of schools remaining after x rounds.
 38. How many schools are left after 3 rounds?
 39. How many rounds will it take to declare a champion?

TRAINING For Exercises 40 and 41, use the following information.

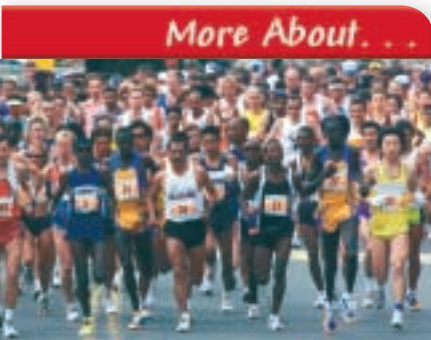
A runner is training for a marathon, running a total of 20 miles per week on a regular basis. She plans to increase the distance $D(x)$ in miles according to the function $D(x) = 20(1.1)^x$, where x represents the number of weeks of training.

40. Copy and complete the table showing the number of miles she plans to run.
 41. The runner's goal is to work up to 50 miles per week. What is the first week that the total will be 50 miles or more?

Week	Distance (miles)
1	
2	
3	
4	

CRITICAL THINKING Describe the graph of each equation as a transformation of the graph of $y = 5^x$.

42. $y = \left(\frac{1}{5}\right)^x$ 43. $y = 5^x + 2$ 44. $y = 5^x - 4$



More About . . .

Training

The first Boston Marathon was held in 1896. The distance of this race was based on the Greek legend that Pheidippides ran 24.8 miles from Marathon to Athens to bring the news of victory over the Persian army.

Source: www.bostonmarathon.org



45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can exponential functions be used in art?

Include the following in your answer:

- the exponential function representing the pliers,
- an explanation of which x and y values are meaningful, and
- the graph of this function.

**Standardized
Test Practice**

A B C D

46. Which function is an exponential function?
- (A) $f(x) = x^2$ (B) $f(x) = 6^x$
(C) $f(x) = x^5$ (D) $f(x) = x^3 + 2x^2 - x + 5$
47. Compare the graphs of $y = 2^x$ and $y = 6^x$.
- (A) The graph of $y = 6^x$ steeper than the graph of $y = 2^x$.
(B) The graph of $y = 2^x$ steeper than the graph of $y = 6^x$.
(C) The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 4 units up.
(D) The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 3 units up.

Maintain Your Skills

Mixed Review Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 10-4)

48. $x^2 - 9x - 36 = 0$ 49. $2t^2 + 3t - 1 = 0$ 50. $5y^2 + 3 = y$

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 10-3)

51. $x^2 - 7x = -10$ 52. $a^2 - 12a = 3$ 53. $t^2 + 6t + 3 = 0$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 9-3)

54. $m^2 - 14m + 40$ 55. $t^2 - 2t + 35$ 56. $z^2 - 5z - 24$

57. Three times one number equals twice a second number. Twice the first number is 3 more than the second number. Find the numbers. (Lesson 7-4)

Solve each inequality. (Lesson 6-1)

58. $x + 7 > 2$ 59. $10 \geq x + 8$ 60. $y - 7 < -12$

**Getting Ready for
the Next Lesson** **PREREQUISITE SKILL** Evaluate $p(1 + r)^t$ for each of the given values. (To review *evaluating expressions with exponents*, see Lesson 1-1.)

61. $p = 5, r = \frac{1}{2}, t = 2$ 62. $p = 300, r = \frac{1}{4}, t = 3$
63. $p = 100, r = 0.2, t = 2$ 64. $p = 6, r = 0.5, t = 3$

Practice Quiz 2

Lessons 10-4 and 10-5

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 10-4)

1. $x^2 + 2x = 35$ 2. $2n^2 - 3n + 5 = 0$ 3. $2v^2 - 4v = 1$

Graph each function. State the y -intercept. (Lesson 10-5)

4. $y = 0.5(4^x)$ 5. $y = 5^x - 4$

What You'll Learn

- Solve problems involving exponential growth.
- Solve problems involving exponential decay.

Vocabulary

- exponential growth
- compound interest
- exponential decay

How

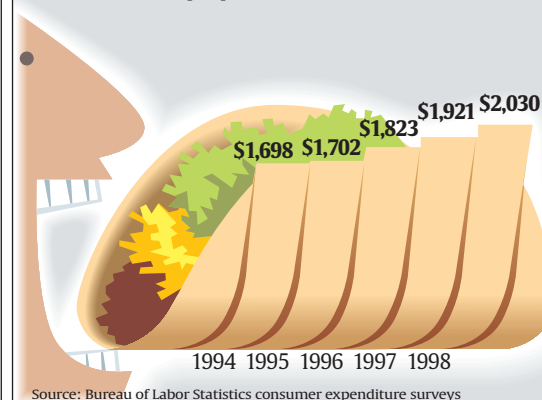
can exponential growth be used to predict future sales?

The graph shows that the average household in the United States has increased its spending for restaurant meals. In fact, the amount grew at an annual rate of about 4.6% between 1994 and 1998. Let y represent the average amount spent on restaurant meals, and let t represent the number of years since 1994. Then the average amount spent on restaurant meals can be modeled by $y = 1698(1 + 0.046)^t$ or $y = 1698(1.046)^t$.

USA TODAY Snapshots®

**Spending more on eating out**

Annual spending on eating out, by year, for an average household of 2.5 people:



EXPONENTIAL GROWTH The equation for the average amount spent on restaurant meals is in the form $y = C(1 + r)^t$. This is the general equation for **exponential growth** in which the initial amount C increases by the same percent over a given period of time.

Key Concept*General Equation for Exponential Growth*

The general equation for exponential growth is $y = C(1 + r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.

Example 1 Exponential Growth

SPORTS In 1971, there were 294,105 females participating in high school sports. Since then, that number has increased an average of 8.5% per year.

- a. Write an equation to represent the number of females participating in high school sports since 1971.

$$y = C(1 + r)^t \quad \text{General equation for exponential growth}$$

$$y = 294,105(1 + 0.085)^t \quad C = 294,105 \text{ and } r = 8.5\% \text{ or } 0.085$$

$$y = 294,105(1.085)^t \quad \text{Simplify.}$$

An equation to represent the number of females participating in high school sports is $y = 294,105(1.085)^t$, where y represents the number of female athletes and t represents the number of years since 1971.

(continued on the next page)



- b. According to the equation, how many females participated in high school sports in the year 2001?

$$y = 294,105(1.085)^t \quad \text{Equation for females participating in sports}$$

$$y = 294,105(1.085)^{30} \quad t = 2001 - 1971 \text{ or } 30$$

$$y \approx 3,399,340 \quad \text{In 2001, about 3,399,340 females participated.}$$

One special application of exponential growth is **compound interest**. The equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the amount of the investment, P is the principal (initial amount of the investment), r represents the annual rate of interest expressed as a decimal, n represents the number of times that the interest is compounded each year, and t represents the number of years that the money is invested.

Example 2 Compound Interest

- HISTORY** Use the information at the left. If the money the Native Americans received for Manhattan had been invested at 6% per year compounded semiannually, how much money would there be in the year 2026?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{Compound interest equation}$$

$$A = 24\left(1 + \frac{0.06}{2}\right)^{2(400)} \quad P = 24, r = 6\% \text{ or } 0.06, n = 2, \text{ and } t = 400$$

$$A = 24(1.03)^{800} \quad \text{Simplify.}$$

$$A \approx 4.47 \times 10^{11} \quad \text{There would be about } \$447,000,000,000.$$

EXPONENTIAL DECAY A variation of the growth equation can be used as the general equation for **exponential decay**. In exponential decay, the original amount decreases by the same percent over a period of time.

Key Concept General Equation for Exponential Decay

The general equation for exponential decay is $y = C(1 - r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of decay expressed as a decimal, and t represents time.

Example 3 Exponential Decay

ENERGY In 1950, the use of coal by residential and commercial users was 114.6 million tons. Many businesses now use cleaner sources of energy. As a result, the use of coal has decreased by 6.6% per year.

- a. Write an equation to represent the use of coal since 1950.

$$y = C(1 - r)^t \quad \text{General equation for exponential decay}$$

$$y = 114.6(1 - 0.066)^t \quad C = 114.6 \text{ and } r = 6.6\% \text{ or } 0.066$$

$$y = 114.6(0.934)^t \quad \text{Simplify.}$$

An equation to represent the use of coal is $y = 114.6(0.934)^t$, where y represents tons of coal used annually and t represents the number of years since 1950.

- b. Estimate the estimated amount of coal that will be used in 2015.

$$y = 114.6(0.934)^t \quad \text{Equation for coal use}$$

$$y = 114.6(0.934)^{65} \quad t = 2015 - 1950 \text{ or } 65$$

$$y \approx 1.35 \quad \text{The amount of coal should be about 1.35 million tons.}$$

More About . . .



History

In 1626, Peter Minuit, governor of the colony of New Netherland, bought the island of Manhattan from the Native Americans for beads, cloth, and trinkets worth 60 Dutch guilders (\$24).

Source: *World Book Encyclopedia*

Sometimes items decrease in value or *depreciate*. For example, most cars and office equipment depreciate as they get older. You can use the exponential decay formula to determine the value of an item at a given time.

Example 4 Depreciation

FARMING A farmer buys a tractor for \$50,000. If the tractor depreciates 10% per year, find the value of the tractor in 7 years.

$$y = C(1 - r)^t \quad \text{General equation for exponential decay}$$

$$y = 50,000(1 - 0.10)^7 \quad C = 50,000, r = 10\% \text{ or } 0.10, \text{ and } t = 7$$

$$y = 50,000(0.90)^7 \quad \text{Simplify.}$$

$$y \approx 23,914.85 \quad \text{Use a calculator.}$$

The tractor will be worth about \$23,914.85 or less than half its original value.

Check for Understanding

Concept Check

1. Explain the difference between *exponential growth* and *exponential decay*.
2. **OPEN ENDED** Write a compound interest problem that could be solved by the equation $A = 500\left(1 + \frac{0.07}{4}\right)^{4(6)}$.
3. Draw a graph representing exponential decay.

Guided Practice

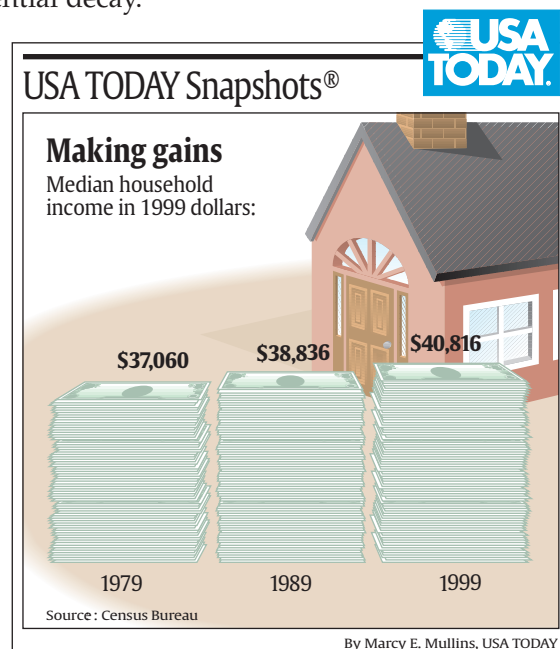
INCOME For Exercises 4 and 5, use the graph at the right and the following information.

The median household income in the United States increased an average of 0.5% each year between 1979 and 1999. Assume this pattern continues.

4. Write an equation for the median household income for t years after 1979.
5. Predict the median household income in 2009.
6. **INVESTMENTS** Determine the amount of an investment if \$400 is invested at an interest rate of 7.25% compounded quarterly for 7 years.

Applications

7. **POPULATION** In 1995, the population of West Virginia reached 1,821,000, its highest in the 20th century. For the next 5 years, its population decreased 0.2% each year. If this trend continues, predict the population of West Virginia in 2010.
8. **TRANSPORTATION** A car sells for \$16,000. If the rate of depreciation is 18%, find the value of the car after 8 years.



Practice and Apply

TECHNOLOGY For Exercises 9 and 10, use the following information.

Computer use around the world has risen 19% annually since 1980.

9. If 18.9 million computers were in use in 1980, write an equation for the number of computers in use for t years after 1980.
10. Predict the number of computers in 2015.



Homework Help

For Exercises	See Examples
9–13, 18 21, 22	1
14, 15	2
16, 17, 25–28	3
19, 20	4

Extra Practice

See page 843.



Grand Canyon

The Grand Canyon National Park covers 1,218,375 acres. It has 38 hiking trails, which cover about 400 miles.

Source: *World Book Encyclopedia*

WEIGHT TRAINING

For Exercises 11 and 12, use the following information. The use of free weights for fitness has increased in popularity. In 1997, there were 43.2 million people who used free weights.

- Assuming the use of free weights increases 6% annually, write an equation for the number of people using free weights t years from 1997.
- Predict the number of people using free weights in 2007.

13. POPULATION The population of Mexico has been increasing at an annual rate of 1.7%. If the population of Mexico was 100,350,000 in the year 2000, predict its population in 2012.

14. INVESTMENTS Determine the amount of an investment if \$500 is invested at an interest rate of 5.75% compounded monthly for 25 years.

15. INVESTMENTS Determine the amount of an investment if \$250 is invested at an interest rate of 10.3% compounded quarterly for 40 years.

16. POPULATION The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2000, its population was 2,405,000. If the trend continues, predict Latvia's population in 2015.

17. MUSIC In 1994, the sales of music cassettes reached its peak at \$2,976,400,000. Since then, cassette sales have been declining. If the annual percent of decrease in sales is 18.6%, predict the sales of cassettes in the year 2009.

18. GRAND CANYON The increase in the number of visitors to the Grand Canyon National Park is similar to an exponential function. If the average visitation has increased 5.63% annually since 1920, use the graph to predict the number of visitors to the park in 2020.

19. BUSINESS A piece of office equipment valued at \$25,000 depreciates at a steady rate of 10% per year. What is the value of the equipment in 8 years?

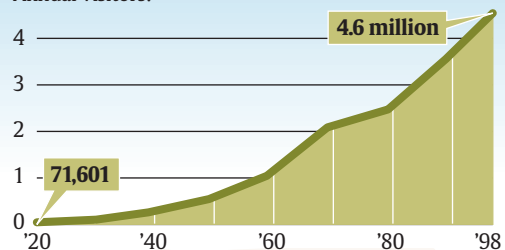
20. TRANSPORTATION A new car costs \$23,000. It is expected to depreciate 12% each year. Find the value of the car in 5 years.

USA TODAY Snapshots®



Grand Canyon Visitors

Annual visitors:



Source: Grand Canyon National Park

By Marcy E. Mullins, USA TODAY

POPULATION

For Exercises 21 and 22, use the following information. Since birth rates are going down and people are living longer, the percent of the population that is 65 years old or older continues to rise. The percent of the U.S. population P that is at least 65 years old can be approximated by the equation $P = 3.86(1.013)^t$, where t represents the number of years since 1900.

- What percent of the population will be 65 years of age or older in the year 2010?
- Predict the year in which people ages 65 or older will represent 20% of the population if this trend continues. (*Hint*: Make a table.)

CRITICAL THINKING Each equation represents an exponential rate of change if t is time in years. Determine whether each equation represents growth or decay. Give the annual rate of change as a percent.

23. $y = 500(1.026^t)$

24. $y = 500(0.761^t)$



ARCHAEOLOGY For Exercises 25–28, use the following information.

The *half-life* of a radioactive element is the time that it takes for one-half a quantity of the element to decay. Carbon-14 is found in all living organisms and has a half-life of 5730 years. Archaeologists use this fact to estimate the age of fossils. Consider an organism with an original Carbon-14 content of 256 grams. The number of grams remaining in the organism's fossil after t years is $256(0.5)^{\frac{t}{5730}}$.

25. If the organism died 5730 years ago, what is the amount of Carbon-14 today?
 26. If the organism died 1000 years ago, what is the amount of Carbon-14 today?
 27. If the organism died 10,000 years ago, what is the amount of Carbon-14 today?
 28. If the fossil has 32 grams of Carbon-14 remaining, how long ago did it live?
 (*Hint:* Make a table.)

29. **RESEARCH** Find the enrollment of your school district each year for the last decade. Find the rate of change from one year to the next. Then, determine the average annual rate of change for those years. Use this information to estimate the enrollment for your school district in ten years.

30. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can exponential growth be used to predict future sales?

Include the following in your answer:

- an explanation of the equation $y = 1698(1 + 0.046)^t$, and
- an estimate of the average family's spending for restaurant meals in 2010.

31. Which equation represents exponential growth?

- (A) $y = 50x^3$ (B) $y = 30x^2 + 10$
 (C) $y = 35(1.05^x)$ (D) $y = 80(0.92^x)$

32. Lorena is investing a \$5000 inheritance from her aunt in a certificate of deposit that matures in 4 years. The interest rate is 8.25% compounded quarterly. What is the balance of the account after 4 years?

- (A) \$5412.50 (B) \$6865.65 (C) \$6908.92 (D) \$6931.53


**Standardized
Test Practice**

(A) (B) (C) (D)

Maintain Your Skills

Mixed Review Graph each function. State the y -intercept. (*Lesson 10-5*)

33. $y = \left(\frac{1}{8}\right)^x$ 34. $y = 2^x - 5$ 35. $y = 4(3^x - 6)$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (*Lesson 10-4*)

36. $m^2 - 9m - 10 = 0$ 37. $2t^2 - 4t = 3$ 38. $7x^2 + 3x + 1 = 0$

Simplify. (*Lesson 8-1*)

39. $m^7(m^3b^2)$ 40. $-3(ax^3y)^2$ 41. $(0.3x^3y^2)^2$

Solve each open sentence. (*Lesson 6-5*)

42. $|7x + 2| = -2$ 43. $|3 - 3x| = 0$ 44. $|t + 4| \geq 3$

45. **SKIING** A course for cross-country skiing is regulated so that the slope of any hill cannot be greater than 0.33. A hill rises 60 meters over a horizontal distance of 250 meters. Does the hill meet the requirements? (*Lesson 5-1*)

**Getting Ready for
the Next Lesson**

PREREQUISITE SKILL Find the next three terms in each arithmetic sequence.
 (*To review arithmetic sequences, see Lesson 4-7.*)

46. 8, 11, 14, 17, ... 47. 7, 4, 1, -2, ... 48. 1.5, 2.6, 3.7, 4.8, ...





Reading Mathematics

Growth and Decay Formulas

Growth and decay problems may be confusing, unless you read them in a simplified, generalized form. The growth and decay formulas that you used in Lesson 10-6 are based on the idea that an initial amount is multiplied by a rate raised to a power of time, which is equivalent to a final amount. If you remember the following formula, all other formulas will be easier to remember.

$$\text{final amount} = \text{initial amount} \cdot \text{rate}^{\text{time}}$$

Below, we will review the general equation for exponential growth to see how it is related to the generalized formula above.

$$\begin{array}{ccccccc} \text{The final} & & \text{an initial} & & \text{the quantity one plus a rate} \\ \text{amount} & \text{equals} & \text{amount} & \text{times} & \text{raised to the power of time.} \\ \hline y & = & C & \cdot & (1 + r)^t \end{array}$$

The only difference from the generalized formula is that rate equals $1 + r$. *Why?*

One represents 100%. If you multiply C by 100%, the final amount is the same as the initial amount. We *add* 1 to the rate r so that the final amount is the initial amount plus the increase.

You can break each growth and decay formula into the following pieces:

- final amount,
- initial amount,
- rate, and
- time.

Reading to Learn

1. Write the general equation for exponential decay. Discuss how it is related to the generalized formula. Why is the rate equal to $1 - r$?
2. Write the formula for compound interest. How is it related to the generalized formula? Why does the rate equal $\left(1 + \frac{r}{n}\right)$? Why does the time equal nt ?
3. Suppose that \$2500 is invested at an annual rate of 6%. If the interest is compounded quarterly, find the value of the account after 5 years.
 - a. Copy the problem and underline all important numerical data.
 - b. Choose the appropriate formula and solve the problem.
4. Angela bought a car for \$18,500. If the rate of depreciation is 11%, find the value of the car in 4 years.
 - a. Copy the problem and underline all important numerical data.
 - b. Choose the appropriate formula and solve the problem.
5. The population of Centerville is increasing at an average annual rate of 3.5%. If its current population is 12,500, predict its population in 5 years.
 - a. Copy the problem and underline all important numerical data.
 - b. Choose the appropriate formula and solve the problem.

What You'll Learn

- Recognize and extend geometric sequences.
- Find geometric means.

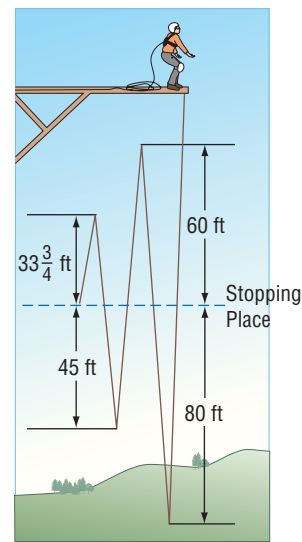
Vocabulary

- geometric sequence
- common ratio
- geometric means

How can a geometric sequence be used to describe a bungee jump?

A thrill ride is set up with a bungee rope that will stretch when a person jumps from the platform. The ride continues as the person bounces back and forth closer to the stopping place of the rope. Each bounce is only $\frac{3}{4}$ as far from the stopping length as the preceding bounce. If the initial drop is 80 feet past the stopping length of the rope, the following table gives the distance of the first four bounces.

Bounce	Distance (ft)
1	80
2	$\frac{3}{4} \cdot 80$ or 60
3	$\frac{3}{4} \cdot 60$ or 45
4	$\frac{3}{4} \cdot 45$ or $33\frac{3}{4}$



GEOMETRIC SEQUENCES The distance of each bounce is found by multiplying the previous term by $\frac{3}{4}$. The successive distances of the bounces is an example of a **geometric sequence**. The number by which each term is multiplied is called the **common ratio**.

Key Concept*Geometric Sequence*

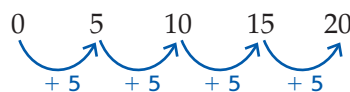
- **Words** A geometric sequence is a sequence in which each term after the nonzero first term is found by multiplying the previous term by a constant called the common ratio r , where $r \neq 0, 1$.
- **Symbols** $a, ar, (ar)r$ or $ar^2, (ar^2)r$ or ar^3, \dots ($a \neq 0; r \neq 0, 1$)
- **Examples** 1, 3, 9, 27, 81, ...

Example 1 Recognize Geometric Sequences

Determine whether each sequence is geometric.

- a. 0, 5, 10, 15, 20, ...

Determine the pattern.

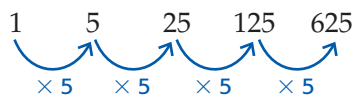


In this sequence, each term is found by adding 5 to the previous term. This sequence is arithmetic, *not* geometric.

Study Tip**Look Back**

To review **arithmetic sequences**, see Lesson 4-7.

b. 1, 5, 25, 125, 625



In this sequence, each term is found by multiplying the previous term times 5. This sequence is geometric.

The common ratio of a geometric sequence can be found by dividing any term by the preceding term.

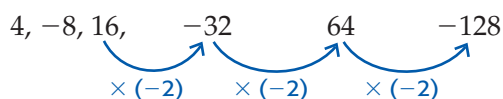
Example 2 Continue Geometric Sequences

Find the next three terms in each geometric sequence.

a. 4, -8, 16, ...

$$\frac{-8}{4} = -2 \quad \text{Divide the second term by the first.}$$

The common factor is -2 . Use this information to find the next three terms.

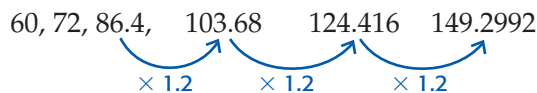


The next three terms are -32 , 64 , and -128 .

b. 60, 72, 86.4, ...

$$\frac{72}{60} = 1.2 \quad \text{Divide the second term by the first.}$$

The common factor is 1.2 . Use this information to find the next three terms.



The next three terms are 103.68 , 124.416 , and 149.2992 .



More About . . .

Geography

Madagascar is a country just east of mainland Africa. It consists of the third largest island in the world and many tiny islands. In 2000, the population of Madagascar was about 15,500,000.

Source: *World Book Encyclopedia*

Example 3 Use Geometric Sequences to Solve a Problem

GEOGRAPHY Madagascar's population has been increasing at an average annual rate of 3%. Use the information at the left to determine the population of Madagascar in 2001, 2002, and 2003.

The population is a geometric sequence in which the first term is 15,500,000 and the common ratio is 1.03.

Year	Population
2000	15,500,000
2001	15,500,000(1.03) or 15,965,000
2002	15,965,000(1.03) or 16,443,950
2003	16,443,950(1.03) or about 16,937,269

The population of Madagascar in the years 2001, 2002, and 2003 will be 15,965,000, 16,443,950, and about 16,937,269, respectively.

As with arithmetic sequences, you can name the terms of a geometric sequence using $a_1, a_2, a_3,$ and so on. Then the n th term is represented as a_n . Each term of a geometric sequence can also be represented using r and its previous term. A third way to represent each term is by using r and the first term a_1 .



Sequence	number	2	6	18	54	...	
	symbols	a_1	a_2	a_3	a_4	...	a_n
Expressed in Terms of r and Previous Term	number	2	$2(3)$	$6(3)$	$18(3)$...	
	symbols	a_1	$a_1 \cdot r$	$a_2 \cdot r$	$a_3 \cdot r$...	$a_{n-1} \cdot r$
Expressed in Terms of r and First Term	number	2 or $2(3^0)$	$2(3)$ or $2(3^1)$	$2(9)$ or $2(3^2)$	$2(27)$ or $2(3^3)$...	
	symbols	$a_1 \cdot r^0$	$a_1 \cdot r^1$	$a_1 \cdot r^2$	$a_1 \cdot r^3$...	$a_1 \cdot r^{n-1}$

The three values in the last column of the table all describe the n th term of a geometric sequence.

Study Tip

Recursive Formulas

When the n th term of a sequence is expressed in terms of the previous term, as in $a_n = a_{n-1} \cdot r$, the formula is called a recursive formula.

Key Concept Formula for the n th Term of a Geometric Sequence

The n th term a_n of a geometric sequence with the first term a_1 and common ratio r is given by $a_n = a_1 \cdot r^{n-1}$.

Example 4 n th Term of a Geometric Sequence

Find the sixth term of a geometric sequence in which $a_1 = 3$ and $r = -5$.

$$a_n = a_1 \cdot r^{n-1} \quad \text{Formula for the } n\text{th term of a geometric sequence}$$

$$a_6 = 3 \cdot (-5)^{6-1} \quad n = 6, a_1 = 3, \text{ and } r = -5$$

$$a_6 = 3 \cdot (-5)^5 \quad 6 - 1 = 5$$

$$a_6 = 3 \cdot (-3125) \quad (-5)^5 = -3125$$

$$a_6 = -9375 \quad 3 \cdot (-3125) = -9375$$

The sixth term of the geometric sequence is -9375 .

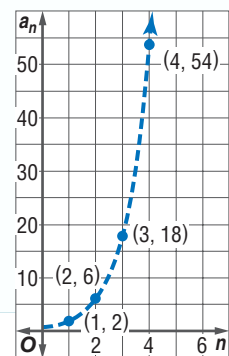
Geometric sequences are related to exponential functions.



Algebra Activity

Graphs of Geometric Sequences

You can graph a geometric sequence by graphing the coordinates (n, a_n) . For example, consider the sequence 2, 6, 18, 54, To graph this sequence, graph the points at $(1, 2)$, $(2, 6)$, $(3, 18)$, and $(4, 54)$. Use a dashed curve to connect the points.



Model

Graph each geometric sequence. Name each common ratio.

- 1, 2, 4, 8, 16, ...
- 1, -2, 4, -8, 16, ...
- 81, 27, 9, 3, 1, ...
- 81, 27, -9, 3, -1, ...
- 0.2, 1, 5, 25, 125, ...
- 0.2, 1, -5, 25, -125, ...

Analyze

- Which graphs appear to be similar to an exponential function?
- Compare and contrast the graphs of geometric sequences with $r > 0$ and $r < 0$.
- Compare the formula for an exponential function $y = c(a^x)$ to the value of the n th term of a geometric sequence.

GEOMETRIC MEANS Missing term(s) between two nonconsecutive terms in a geometric sequence are called **geometric means**. In the sequence 100, 20, 4, ..., the geometric mean between 100 and 4 is 20. You can use the formula for the n th term of a geometric sequence to find a geometric mean.

Example 5 Find Geometric Means

Find the geometric mean in the sequence 2, ____, 18.

In the sequence, $a_1 = 2$ and $a_3 = 18$. To find a_2 , you must first find r .

$a_n = a_1 \cdot r^{n-1}$ Formula for the n th term of a geometric sequence

$a_3 = a_1 \cdot r^{3-1}$ $n = 3$

$18 = 2 \cdot r^2$ $a_3 = 18$ and $a_1 = 2$

$\frac{18}{2} = \frac{2r^2}{2}$ Divide each side by 2.

$9 = r^2$ Simplify.

$\pm 3 = r$ Take the square root of each side.

If $r = 3$, the geometric mean is $2(3)$ or 6. If $r = -3$, the geometric mean is $2(-3)$ or -6 . Therefore, the geometric mean is 6 or -6 .

Check for Understanding

Concept Check

1. Compare and contrast an arithmetic sequence and a geometric sequence.
2. Explain why the definition of a geometric sequence restricts the values of the common ratio to numbers other than 0 and 1.
3. **OPEN ENDED** Give an example of a sequence that is neither arithmetic nor geometric.

Guided Practice

Determine whether each sequence is geometric.

4. 5, 15, 45, 135, ... 5. 56, -28 , 14, -7 , ... 6. 25, 20, 15, 10, ...

Find the next three terms in each geometric sequence.

7. 5, 20, 80, 320, ... 8. 176, -88 , 44, -22 , ... 9. -8 , 12, -18 , 27, ...

Find the n th term of each geometric sequence.

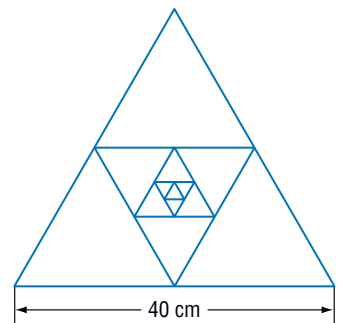
10. $a_1 = 3$, $n = 5$, $r = 4$ 11. $a_1 = -1$, $n = 6$, $r = 2$ 12. $a_1 = 4$, $n = 7$, $r = -3$

Find the geometric means in each sequence.

13. 7, ____, 28 14. 48, ____, 3 15. -4 , ____, -100

Application

16. **GEOMETRY** Consider the inscribed equilateral triangles at the right. The perimeter of each triangle is one-half of the perimeter of the next larger triangle. What is the perimeter of the smallest triangle?



Practice and Apply

Homework Help

For Exercises	See Examples
17–24	1
25–34	2
35–42	4
43–54	5
55–62	3

Extra Practice

See page 843.

Determine whether each sequence is geometric.

17. 2, 6, 18, 54, ... 18. 7, 17, 27, 37, ... 19. -19, -16, -13, -10, ...
 20. 640, 160, 40, 10, ... 21. 36, 25, 16, 9, ... 22. -567, -189, -63, -21, ...
 23. 20, -90, 405, -1822.5, ... 24. -50, 110, -242, 532.4, ...

Find the next three terms in each geometric sequence.

25. 1, -4, 16, -64, ... 26. -1, -6, -36, -216, ... 27. 1024, 512, 256, 128, ...
 28. 224, 112, 56, 28, ... 29. -80, 20, -5, 1.25, ... 30. 10,000, -200, 4, -0.08, ...
 31. $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \dots$ 32. $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \dots$

33. **GEOMETRY** A rectangle is 6 inches by 8 inches. The rectangle is cut in half, and one half is discarded. The remaining rectangle is cut in half, and one half is discarded. This is repeated twice. List the areas of the five rectangles formed.
 34. **GEOMETRY** To bisect an angle means to cut it into two angles with the same measure. Suppose a 160° angle is bisected. Then one of the new angles is bisected. This is repeated twice. List the measures of the four sizes of angles.

Find the n th term of each geometric sequence.

35. $a_1 = 5, n = 7, r = 2$ 36. $a_1 = 4, n = 5, r = 3$ 37. $a_1 = -2, n = 4, r = -5$
 38. $a_1 = 3, n = 6, r = -4$ 39. $a_1 = -8, n = 3, r = 6$ 40. $a_1 = -10, n = 8, r = 2$
 41. $a_1 = 300, n = 10, r = 0.5$ 42. $a_1 = 14, n = 6, r = 1.5$

Find the geometric means in each sequence.

43. 5, ____, 20 44. 6, ____, 54 45. -9, ____, -225
 46. -5, ____, -80 47. 128, ____, 8 48. 180, ____, 5
 49. -2, ____, -98 50. -6, ____, -384 51. 7, ____, 1.75
 52. 3, ____, 0.75 53. $\frac{3}{5}, \text{---}, \frac{3}{20}$ 54. $\frac{2}{5}, \text{---}, \frac{2}{45}$

55. A ball is thrown vertically. It is allowed to return to the ground and rebound without interference. If each rebound is 60% of the previous height, give the heights of the three rebounds after the initial rebound of 10 meters.

QUIZ GAMES For Exercises 56 and 57, use the following information.

Radio station WXYZ has a special game for its listeners. A trivia question is asked, and the player scores 10 points for the first correct answer. Every correct answer after that doubles the player's score.

56. List the scores after each of the first 6 correct answers.
 57. Suppose the player needs to answer the question worth more than a million points to win the grand prize of a car. How many questions must be answered correctly in order to earn the car?

POLLUTION For Exercise 58–60, use the following information.

A lake was closed because of an accidental pesticide spill. The concentration of the pesticide after the spill was 848 parts per million. Each day the water is tested, and the amount of pesticide is found to be about 75% of what was there the day before.

58. List the level of pesticides in the water during the first week.
 59. If a safe level of pesticides is considered to be 12 parts per million or less, when will the lake be considered safe?
 60. Do you think the lake will ever be completely free of the pesticide? Explain.



Pollution

On March 23, 1989, 250,000 barrels of oil were spilled affecting 1300 miles of Alaskan coastline. This was the largest oil spill in the United States.

Source: www.oilspill.state.ak.us



CRITICAL THINKING For Exercises 61 and 62, suppose a sequence is geometric.

61. If each term of the sequence is multiplied by the same nonzero real number, is the new sequence *always*, *sometimes*, or *never* a geometric sequence?
62. If the same nonzero number is added to each term of the sequence, is the new sequence *always*, *sometimes*, or *never* a geometric sequence?
63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a geometric sequence be used to describe a bungee jump?

Include the following in your answer:

- an explanation of how to determine the tenth term in the sequence, and
- the number of rebounds the first time the distance from the stopping place is less than one foot, which would trigger the end of the ride.

**Standardized
Test Practice**

64. Which number is next in the geometric sequence 40, 100, 250, 625, ... ?
(A) 900 (B) 1250 (C) 1562.5 (D) 1875
65. **GRID IN** Find the next term in the following geometric sequence.
343, 49, 7, 1, ...

**Extending
the Lesson**

- For Exercises 66–68, consider the n th term of the sequence $2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \dots$.
66. As n approaches infinity, what value will the n th term approach?
67. In mathematics, a **limit** is a number that something approaches, but never reaches. What would you consider the limit of the values of the sequence?
68. If n approaches infinity, how is the n th term of a geometric sequences where $0 < r < 1$ different than the n th term of a geometric sequences where $r > 1$?

Maintain Your Skills

Mixed Review

69. **INVESTMENTS** Determine the value of an investment if \$1500 is invested at an interest rate of 6.5% compounded monthly for 3 years. (Lesson 10-6)

Determine whether the data in each table display exponential behavior. Explain why or why not. (Lesson 10-5)

70.

x	3	5	7	9
y	10	12	14	16

71.

x	2	5	8	11
y	0.5	1.5	4.5	13.5

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 9-4)

72. $7a^2 + 22a + 3$

73. $2x^2 - 5x - 12$

74. $3c^2 - 3c - 5$

WebQuest Internet Project

Pluto Is Falling from Status as a Distant Planet

It is time to complete your project. Use the information and data you have gathered about the solar system to prepare a brochure, poster, or Web page. Be sure to include the three graphs, tables, diagrams, or calculations in the presentation.



www.algebra1.com/webquest



Algebra Activity

A Follow-Up of Lesson 10-7

Investigating Rates of Change

Collect the Data

- The Richter scale is used to measure the force of an earthquake. The table below shows the increase in magnitude for the values on the Richter scale.

Richter Number (x)	Increase in Magnitude (y)	Rate of Change (slope)
1	1	—
2	10	9
3	100	
4	1000	
5	10,000	
6	100,000	
7	1,000,000	

Source: *The New York Public Library Science Desk Reference*

- On grid paper, plot the ordered pairs (Richter number, increase in magnitude).
- Copy the table for the Richter scale and fill in the rate of change from one value to the next. For example, the rate of change for (1, 1) and (2, 10) is $\frac{10 - 1}{2 - 1}$ or 9.

Analyze the Data

- Describe the graph you made of the Richter scale data.
- Is the rate of change between any two points the same?

Make a Conjecture

- Can the data be represented by a linear equation? Why or why not?
- Describe the pattern shown in the rates of change in Column 3.

Extend the Investigation

- Use a graphing calculator or graphing software to find a regression equation for the Richter scale data. (*Hint:* If you are using the TI-83 Plus, use ExpReg.)
- Graph the following set of data that shows the amount of energy released for each Richter scale value. Describe the graph. Fill in the third column and describe the rates of change. Find a regression equation for this set of data.

Richter Number (x)	Energy Released (y)	Rate of Change (slope)
1	0.00017 metric ton	
2	0.006 metric ton	
3	0.179 metric ton	
4	5 metric tons	
5	179 metric tons	
6	5643 metric tons	
7	179,100 metric tons	

Source: *The New York Public Library Science Desk Reference*

Vocabulary and Concept Check

axis of symmetry (p. 525)

common ratio (p. 567)

completing the square (p. 539)

compound interest (p. 562)

discriminant (p. 548)

exponential decay (p. 562)

exponential function (p. 554)

exponential growth (p. 561)

geometric means (p. 570)

geometric sequence (p. 567)

maximum (p. 525)

minimum (p. 525)

parabola (p. 524)

quadratic equation (p. 533)

Quadratic Formula (p. 546)

quadratic function (p. 524)

roots (p. 533)

symmetry (p. 525)

vertex (p. 525)

zeros (p. 533)

Choose the letter of the term that best matches each equation or phrase.

1. $y = C(1 + r)^t$

2. $f(x) = ax^2 + bx + c$

3. a geometric property of parabolas

4. $x = -\frac{b}{2a}$

5. $y = a^x$

6. maximum or minimum point of a parabola

7. $y = C(1 - r)^t$

8. solutions of a quadratic equation

9. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

10. the graph of a quadratic function

a. equation of axis of symmetry

b. exponential decay equation

c. exponential function

d. exponential growth equation

e. parabola

f. Quadratic Formula

g. quadratic function

h. roots

i. symmetry

j. vertex

Lesson-by-Lesson Review

10-1 Graphing Quadratic Functions

See pages
524–530.

Concept Summary

- The standard form of a quadratic function is $y = ax^2 + bx + c$.
- Complete a table of values to graph a quadratic function.
- The equation of the axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.
- The vertex of a parabola is on the axis of symmetry.

Example

Consider the graph of $y = x^2 - 8x + 12$.

a. Write the equation of the axis of symmetry.

In the equation $y = x^2 - 8x + 12$, $a = 1$ and $b = -8$. Substitute these values into the equation of the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

$$= -\frac{-8}{2(1)} \text{ or } 4 \quad a = 1 \text{ and } b = -8$$

The equation of the axis of symmetry is $x = 4$.

- b. Find the coordinates of the vertex of the graph.

The x -coordinate of the vertex is 4.

$$y = x^2 - 8x + 12 \quad \text{Original equation}$$

$$y = (4)^2 - 8(4) + 12 \quad x = 4$$

$$y = 16 - 32 + 12 \quad \text{Simplify.}$$

$$y = -4 \quad \text{The coordinates of the vertex are } (4, -4).$$

Exercises Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. See Example 3 on pages 526 and 527.

11. $y = x^2 + 2x$

12. $y = -3x^2 + 4$

13. $y = x^2 - 3x - 4$

14. $y = 3x^2 + 6x - 17$

15. $y = -2x^2 + 1$

16. $y = -x^2 - 3x$

10-2 Solving Quadratic Equations by Graphing

See pages
533–538.

Concept Summary

- The roots of a quadratic equation are the x -intercepts of the related quadratic function.

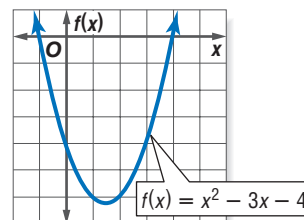
Example

Solve $x^2 - 3x - 4 = 0$ by graphing.

Graph the related function

$$f(x) = x^2 - 3x - 4.$$

The x -intercepts are -1 and 4 . Therefore, the solutions are -1 , and 4 .



Exercises Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

See Examples 1–4 on pages 533–535.

17. $x^2 - x - 12 = 0$

18. $x^2 + 6x + 9 = 0$

19. $x^2 + 4x - 3 = 0$

20. $2x^2 - 5x + 4 = 0$

21. $x^2 - 10x = -21$

22. $6x^2 - 13x = 15$

10-3 Solving Quadratic Equations by Completing the Square

See pages
539–544.

Concept Summary

- Complete the square to make a quadratic expression a perfect square.
- Use the following steps to complete the square of $x^2 + bx$.

Step 1 Find $\frac{1}{2}$ of b , the coefficient of x .

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$, the original expression.

Example

Solve $y^2 + 6y + 2 = 0$ by completing the square. Round to the nearest tenth if necessary.

$$y^2 + 6y + 2 = 0$$

Original equation

$$y^2 + 6y + 2 - 2 = 0 - 2$$

Subtract 2 from each side.

$$y^2 + 6y = -2$$

Simplify.

$$y^2 + 6y + 9 = -2 + 9$$

Since $(\frac{6}{2})^2 = 9$, add 9 to each side.

$$(y + 3)^2 = 7$$

Factor $y^2 + 6y + 9$.

$$y + 3 = \pm\sqrt{7}$$

Take the square root of each side.

$$y + 3 - 3 = \pm\sqrt{7} - 3$$

Subtract 3 from each side.

$$y = -3 \pm \sqrt{7}$$

Simplify.

Use a calculator to evaluate each value of y .

$$y = -3 + \sqrt{7} \quad \text{or} \quad y = -3 - \sqrt{7}$$

$$y \approx -0.4 \quad y \approx -5.6$$

The solution set is $\{-5.6, -0.4\}$.

Exercises Solve each equation by completing the square. Round to the nearest tenth if necessary. See Example 3 on pages 540 and 541.

23. $-3x^2 + 4 = 0$

24. $x^2 - 16x + 32 = 0$

25. $m^2 - 7m = 5$

26. $4a^2 + 16a + 15 = 0$

27. $\frac{1}{2}y^2 + 2y - 1 = 0$

28. $n^2 - 3n + \frac{5}{4} = 0$

10-4

See pages
546–552.

Solving Quadratic Equations by Using the Quadratic Formula

Concept Summary

- The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example

Solve $2x^2 + 7x - 15 = 0$ by using the Quadratic Formula.

For this equation, $a = 2$, $b = 7$, and $c = -15$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$$

$a = 2$, $b = 7$, and $c = -15$

$$x = \frac{-7 \pm \sqrt{169}}{4}$$

Simplify.

$$x = \frac{-7 + 13}{4} \quad \text{or} \quad x = \frac{-7 - 13}{4}$$

$$x = 1\frac{1}{2}$$

$$x = -5$$

The solution set is $\{-5, 1\frac{1}{2}\}$.

Exercises Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. See Examples 1 and 2 on pages 546 and 547.

29. $x^2 - 8x = 20$

30. $r^2 + 10r + 9 = 0$

31. $4p^2 + 4p = 15$

32. $2y^2 + 3 = -8y$

33. $2d^2 + 8d + 3 = 3$

34. $21a^2 + 5a - 7 = 0$

10-5 Exponential Functions

See pages 554–560.

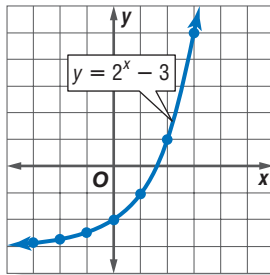
Concept Summary

- An exponential function is a function that can be described by the equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

Example

Graph $y = 2^x - 3$. State the y -intercept.

x	y
-3	-2.875
-2	-2.75
-1	-2.5
0	-2
1	-1
2	1
3	5



Graph the ordered pairs and connect the points with a smooth curve. The y -intercept is -2 .

Exercises Graph each function. State the y -intercept.

See Examples 1 and 2 on page 555.

35. $y = 3^x + 6$

36. $y = 3^{x+2}$

37. $y = 2\left(\frac{1}{2}\right)^x$

10-6 Growth and Decay

See pages 561–565.

Concept Summary

- Exponential Growth: $y = C(1 + r)^t$, where y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.
- Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the amount of the investment, P represents the principal, r represents the annual rate of interest expressed as a decimal, n represents the number of times that the interest is compounded each year, and t represents the number of years that the money is invested.
- Exponential Decay: $y = C(1 - r)^t$, where y represents the final amount, C represents the initial amount, r represents the rate of decay expressed as a decimal, and t represents time.

Example

Find the final amount of an investment if \$1500 is invested at an interest rate of 7.5% compounded quarterly for 10 years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{Compound interest equation}$$

$$A = 1500\left(1 + \frac{0.075}{4}\right)^{4 \cdot 10} \quad P = 1500, r = 7.5\% \text{ or } 0.075, n = 4, \text{ and } t = 10$$

$$A \approx 3153.52 \quad \text{Simplify.}$$

The final amount in the account is about \$3153.52.

Exercises Determine the final amount for each investment.

See Example 2 on page 562.

	Principal	Annual Interest Rate	Time	Type of Compounding
38.	\$2000	8%	8 years	quarterly
39.	\$5500	5.25%	15 years	monthly
40.	\$15,000	7.5%	25 years	monthly
41.	\$500	9.75%	40 years	daily

10-7 Geometric Sequences

See pages 567–572.

Concept Summary

- A geometric sequence is a sequence in which each term after the nonzero first term is found by multiplying the previous term by a constant called the common ratio r , where $r \neq 0$ or 1.
- The n th term a_n of a geometric sequence with the first term a_1 , and a common ratio r is given by $a_n = a_1 \cdot r^{n-1}$.

Example

Find the next three terms in the geometric sequence 7.5, 15, 30,

$$\frac{15}{7.5} = 2 \quad \text{Divide the second term by the first.}$$

The common ratio is 2. Find the next three terms.

$$7.5, 15, 30, \begin{array}{c} \text{60,} \\ \text{120,} \\ \text{240} \end{array}$$

$\times 2 \quad \times 2 \quad \times 2$

The next three terms are 60, 120, and 240.

Exercises Find the n th term of each geometric sequence.

See Example 4 on page 569.

42. $a_1 = 2, n = 5, r = 2$ 43. $a_1 = 7, n = 4, r = \frac{2}{3}$ 44. $a_1 = 243, n = 5, r = -\frac{1}{3}$

Find the geometric means in each sequence. See Example 5 on page 570.

45. 5, _____, 20 46. -12, _____, -48 47. 1, _____, $\frac{1}{4}$

Vocabulary and Concepts

Choose the letter of the term that matches each formula.

1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $y = C(1 + r)^t$

3. $y = C(1 - r)^t$

- a. exponential decay equation
 b. exponential growth equation
 c. Quadratic Formula

Skills and Applications

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

4. $y = x^2 - 4x + 13$

5. $y = -3x^2 - 6x + 4$

6. $y = 2x^2 + 3$

7. $y = -1(x - 2)^2 + 1$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

8. $x^2 - 2x + 2 = 0$

9. $x^2 + 6x = -7$

10. $x^2 + 24x + 144 = 0$

11. $2x^2 - 8x = 42$

Solve each equation. Round to the nearest tenth if necessary.

12. $x^2 + 7x + 6 = 0$

13. $2x^2 - 5x - 12 = 0$

14. $6m^2 + 7n = 20$

15. $3k^2 + 2k = 5$

16. $y^2 - \frac{3}{5}y + \frac{2}{25} = 0$

17. $-3x^2 + 5 = 14x$

18. $z^2 - 13z = 32$

19. $3x^2 + 4a = 8$

20. $7m^2 = m + 5$

Graph each function. State the y -intercept.

21. $y = \left(\frac{1}{2}\right)^x$

22. $y = 4 \cdot 2^x$

23. $y = \left(\frac{1}{3}\right)^x - 3$

Find the n th term of each geometric sequence.

24. $a_1 = 12, n = 6, r = 2$

25. $a_1 = 20, n = 4, r = 3$

Find the geometric means in each sequence.

26. 7, _____, 63

27. $-\frac{1}{3}, \text{_____, } -12$

28. **CARS** Ley needs to replace her car. If she leases a car, she will pay \$410 a month for 2 years and then has the option to buy the car for \$14,458. The current price of the car is \$17,369. If the car depreciates at 16% per year, how will the depreciated price compare with the buyout price of the lease?

29. **FINANCE** Find the total amount after \$1500 is invested for 10 years at a rate of 6%, compounded quarterly.

30. **STANDARDIZED TEST PRACTICE** Which value is the next value in the pattern $-4, 12, -36, 108, \dots$?

(A) -324

(B) 324

(C) -432

(D) 432

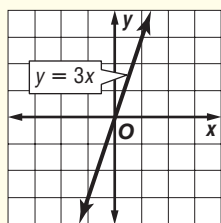


Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The graph of $y = 3x$ is shown. If the line is translated 2 units down, which equation will describe the new line? (Lesson 4-2)

- (A) $y = -6x$
 (B) $y = 3x - 2$
 (C) $y = 3x + 2$
 (D) $y = 3(x - 2)$



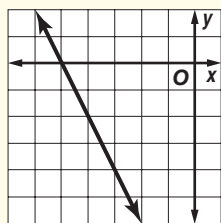
2. Suppose a varies directly as b , and $a = 21$ when $b = 6$. Find a when $b = 28$.

(Lesson 5-2)

- (A) 4.5 (B) 8
 (C) 98 (D) 126

3. Which equation is represented by the graph? (Lesson 5-5)

- (A) $y = -2x - 10$
 (B) $y = -2x - 5$
 (C) $y = 2x + 10$
 (D) $y = 2x - 5$



4. At a farm market, apples cost 20¢ each and grapefruit cost 25¢ each. A shopper bought twice as many apples as grapefruit and spent a total of \$1.95. How many apples did he buy? (Lesson 7-2)

- (A) 3 (B) 4 (C) 5 (D) 6

5. A rectangle has a length of $2x + 3$ and a width of $2x - 6$. Which expression describes the area of the rectangle? (Lesson 8-7)

- (A) $4x - 3$ (B) $4x^2 - 18$
 (C) $4x^2 - 6x - 18$ (D) $4x^2 + 18x - 18$

6. The solution set for the equation $x^2 + x - 12 = 0$ is (Lesson 9-3)

- (A) $\{-4, -3\}$. (B) $\{-4, 3\}$.
 (C) $\{4, -3\}$. (D) $\{4, 3\}$.

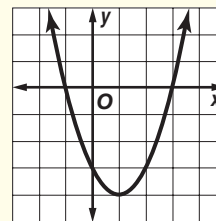
7. Which equation best represents the data in the table? (Lesson 10-1)

x	y
-3	0
-1	8
0	9
2	5
3	0
4	-7

- (A) $y = -x^2 + 3$ (B) $y = -x^2 + 9$
 (C) $y = x^2 - 3$ (D) $y = x^2 + 9$

8. Which equation best represents the parabola graphed below? (Lesson 10-1)

- (A) $y = x^2 - 2x - 4$
 (B) $y = x^2 - 2x - 3$
 (C) $y = x^2 + 2x - 3$
 (D) $y = x^2 + 2x + 3$



9. At which points does the graph of $f(x) = 2x^2 + 8x + 6$ intersect the x -axis? (Lesson 10-2)

- (A) $(-3, 0)$ and $(-2, 0)$
 (B) $(-3, 0)$ and $(-1, 0)$
 (C) $(1, 0)$ and $(3, 0)$
 (D) $(2, 0)$ and $(3, 0)$

The Princeton Review

Test-Taking Tip

Questions 1 and 7 Sketching the graph of a function or a transformation may help you see which answer choice is correct.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

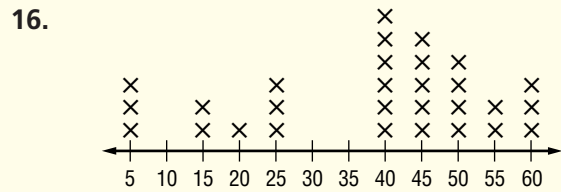
- Monica earned \$18.50, \$23.00, and \$15.00 mowing lawns for 3 consecutive weeks. She wanted to earn an average of at least \$18 per week. What is the minimum she should earn during the 4th week to meet her goal? (Lesson 3-4)
- Write an equation in slope-intercept form of the line that is perpendicular to the line represented by $8x - 4y + 9 = 0$ and passes through the point at $(2, 3)$. (Lesson 5-6)
- If $5a + 4b = 25$ and $3a - 8b = 41$, solve for a and b . (Lesson 7-4)
- Complete the square of $x^2 + 4x - 5$ by finding numbers h and k such that $x^2 + 4x - 5 = (x + h)^2 + k$. (Lesson 10-2)
- At how many points does the graph of $y = 6x^2 + 11x + 4$ intersect the x -axis? (Lesson 10-3)
- The length and width of a rectangle that measures 8 inches by 6 inches are both increased by the same amount. The area of the larger rectangle is twice the area of the original rectangle. How much was added to each dimension of the original rectangle? Round to the nearest hundredth of an inch. (Lesson 10-4)

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- (A) the quantity in Column A is greater,
- (B) the quantity in Column B is greater,
- (C) the two quantities are equal, or
- (D) the relationship cannot be determined from the information given.

Column A	Column B
----------	----------



the mean of the data in the line plot	the median of the data in the line plot
---------------------------------------	---

(Lesson 2-5)

17. the solution of $-6p = -12$	the solution of $10q = 5$
---------------------------------	---------------------------

(Lesson 3-3)

18. 5.3×10^3	53,000
-----------------------	--------

(Lesson 8-3)

19. the 14th term of $-2, -4, -8, \dots$	the 14th term of $2, -4, 8, \dots$
--	------------------------------------

(Lesson 10-7)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

- Analyze the graph of $y = -4x^2 + 8x - \frac{15}{4}$. (Lessons 10-1, 10-3)
 - Show that the equation $-4x^2 + 8x - \frac{15}{4} = -4(x - 1)^2 + \frac{1}{4}$ is always true by expanding the right side.
 - Find the equation of the axis of symmetry of the graph of $y = -4x^2 + 8x - \frac{15}{4}$.
 - Does the parabola open upward or downward? Explain how you determined this.
 - Find the values of x , if any, where the graph crosses the x -axis. Write as rational numbers.
 - Find the coordinates of the maximum or minimum point on this parabola.
 - Sketch the graph of the equation. Label the maximum or minimum point and the roots.